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iii BLOCK V: ADVANCED STATISTICS There are some statistical indicators which are used to indicate the state of economy such as index numbers. Advanced statistical concepts such as index numbers, simulation and linear programming are discussed in this block. Simulation can be used in decision making situations. Linear programming is an operations research technique widely used in business problem solving. This technique can be used to maximize a linear function under certain linear constraints. Linear programming is useful to solve problems in finance, budgeting and investments. There are many applications of linear programming in business and management. Unit-13 Index Numbers explains about index numbers, which are today one of the most widely used statistical indicators. Generally used to indicate the state of the economy, index numbers are aptly called, 'barometers of economic activity'.

Index number is a statistical measure designed to show changes in a variable or a group or related variables with respect to time, geographic location or other characteristics such as income, profession, etc.

The topics Constructing Index Numbers, Average of Relatives Methods, Value Index Numbers, Chain Index Numbers, and Test for Consistency are also discussed in the unit. Unit-14 Simulation deals with decisions taken with simulation process. The future is uncertain, and when decisions are to be taken under conditions of uncertainty, simulation can be used. Simulation as a quantitative method requires the setting up of a mathematical model which would represent the interrelationships between the variables involved in the actual situation in which a decision is to be taken. Then, a number of trails or experiments are conducted with the model to determine the results that can be expected when the variables assume various values. Unit-15 Linear Programming explains the graphical methods and simplex methods of linear programming. Post optimality analysis and duality are also discussed in the unit. A brief review of linear functions is also provided. Several examples with explanation are also given in the unit.

Unit 13 Index Numbers Structure 13.1 Introduction 13.2 Objectives 13.3 Uses of Index Numbers 13.4 Problems Related to Index Numbers 13.5 Types of Index Numbers 13.6 Construction of Index Numbers 13.7 Aggregate Index Numbers 13.8 Unweighted Average of Relatives Method 13.9 Weighted Average of Relatives Method 13.10 Chain Index Number 13.11 Tests for Consistency 13.12

Summary 13.13 Glossary 13.14 Suggested Readings/Reference Material 13.15 Self-Assessment Questions 13.16 Answers to Check Your Progress Questions 13.1 Introduction In the previous unit,

an attempt has been made to give an overview of statistical software – SPSS and SAS. Here, one statistical tool has been used to measure the changes in a variable or a group or related variables with respect to time. Index numbers show how data changes over a period of time relative to a fixed point called base period. They are one of the popular statistical tools to indicate the state of economy. They play a vital role in expressing various economic and fundamental activities over time. In this unit, you will learn the various types of index numbers and their uses. 13.2 Objectives After going through the unit, you should be able to: ? Define index numbers and explain the uses and problems while constructing index numbers; ? State the different types of index numbers; and ? Analyze the procedure to construct index numbers.

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Uses of Index Numbers By definition, an index number is a statistical measure designed to show changes in a variable or a group or related variables with respect to time, geographic location or other characteristics such as income, profession, etc.

Index number is calculated as a ratio of the current value to a base value and expressed as a percentage. It must be clearly understood that the index number for the base year is always 100. An index number is commonly referred to as an index. Generally index numbers were used to study the price level over a period of time. But now, they are extensively used in the field of economics and business to compare data relating to sales, revenue, cost and other such variables. Some of the popular uses of index numbers are summarized below: i. Establishes Trends: Index numbers are said to be the "Barometer" of economy, as they are used to represent various economic indicators and also to establish trends. ii. Framework for Decision-making: Index numbers help governments and businesses in decision-making. For example, using the Consumer Price Index (CPI) and Wholesale Price Index (WPI) values, government and business entities take decisions about the dearness allowance and other wage agreements for their employees. iii. Determines Purchasing Power of Money: Purchasing power of rupee is often determined by the consumer price index. For example, CPI for employees rose from 100 in 2016 to 123 in August 2021. We can say that the real purchasing power of the rupee is 123/100 = 0.81. iv. Forecast Future Events: Index numbers study the relative change in economic phenomenon and hence they help in setting trends in time series data. v. Calculating Real National Income: Index numbers are used to know the real income level of an economy. The adjusted net income with respect to the present price or constant price for a given base year is known as net national product. 13.4 Problems Related to Index Numbers Though index numbers are very useful statistical indicators, the user may face several problems while using the index numbers. Several things can distort index numbers. The most common causes of distortions are: ? Difficulty in Finding Suitable Data: Sometimes there is a difficulty in finding suitable data to compute an index. Suppose a sales manager of a Refrigerator Company is interested in computing an index describing the seasonal variation in the sales of the company's REFRIGERATORS. If the sales are reported only on an annual basis, he would be unable to determine the seasonal sales pattern.

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Incomparability of Indices: Incomparability of indices occurs when attempts are made to compare one index with another after there has been a basic change in what is being measured. If price indices of refrigerators from 2010 to 2020 are compared, one may find that prices have increased substantially. However, this comparison does not take into consideration the technological advances achieved in the quality of refrigerators over the time period under consideration. ? Inappropriate Weighting of Factors: Inappropriate weighting of factors can also distort an index. In developing a composite index, such as the Consumer Price Index, changes in some variables must be considered to be more important than changes in others. ? Choice of Base: Distortion of index numbers also occurs when an improper base is selected. Sometimes a firm selects a base that automatically leads to a result that is in its own interest and proves its initial assumption. Therefore, one must always consider how and why the base period was selected before one accepts a claim based on the result of comparing index numbers. ? Choice of Methods: Because every method has merits and demerits, sometimes it is difficult to select the appropriate methods. 13.5 Types of Index Numbers There are various types of index numbers; of them, price index, quantity index and value index are the three principal ones. Price Index: A price index compares changes in price from one period to another.

Consider the following table: Table 13.1: Prices of Different Articles during Three Years Item/Price Year 1 Year 2 Year 3 Egg (per dozen) 5 8 12 Milk (per litre) 10 14 18 Pulse (per kg) 20 24 32 Rice (per kg) 8 12 18 Sugar (per kg) 8 11 17 Total 51 69 97 Price index for Year 2 and Year 3, if we consider Year 1 as base year will be: The Wholesale Price Index, Consumer Price Index are some of the popularly used price indices.

Block V: Advanced Statistics 4 Quantity Index: Sometimes a researcher is interested in change in quantities than the change in price. For example, while studying consumption a researcher might be interested in quantity. Quantity index measures the changes in quantity from one period to another. Index of Industrial Production (IIP) is a popular quantity index and measures the increase or decrease in the level of industrial production in a given period compared to some base period.

Table 13.2: Quantities of Different Articles during Three Years Item/Quantities (1000) Year1 Year2 Year3 Egg (dozen) 50 55 75 Milk (per litre) 25 28 35 Pulse (kg) 90 100 125 Rice (kg) 215 250 300 Sugar (kg) 120 150 200 Total 500 583 735 Quantity index for Year2 and Year3, if we consider Year1 as base year will be: Value Index: Value Index combines price and quantity changes to present a more spatial comparison. It measures change in net monetary worth. Value index is a useful index to measure changes in sales, inventory, etc. 13.6 Construction of Index Numbers Aggregates method and average of relative method help construct index numbers. Other two methods are weighted method and unweighted method.

Figure 13.1: Flow Chart of Index Numbering

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Aggregate Index Numbers In this section, we will discuss the aggregate method of constructing Index Number. In this method, we have unweighted aggregates index and the weighted aggregates index. 13.7.1 Unweighted Aggregates Index Unweighted aggregates price method is a useful tool for the conversion of price, costs, quantities for different time periods into a number scale with the base year equaling 100%. The above discussed price and quantity index are the examples of unweighted aggregated index number. In general, unweighted aggregates index number is calculated by adding together the current year's/given year's elements and then dividing the result by the sum of the same elements during the base period. Unweighted Aggregates Price Index = 0 1
P P ? ? x 100 Where, 1 P? =
Sum of all elements in the composite for current year 0 P? = Sum of all elements in the composite for base year. Unweighted Aggregates Index is the simplest method of constructing index numbers but it does not reflect the reality since it fails to assign greater weight to high consumption goods over low consumption goods. In other words, the price changes are not linked to any usage/consumption levels. 13.7.2 Weighted Aggregates Index Weighted Aggregates index attaches weights according to their significance and hence is preferred to the unweighted index. Defining weight is the main step in this process. There are various methods of assigning weights to an index. Some of them are as follows: ? Laspeyre's Method ? Paasche's Method ? Fixed-Weight Aggregates Method ? Fisher's Ideal Method ? Marshall-Edgeworth Method. LASPEYRE'S METHOD Laspeyre's method is a weighted aggregate index computed by taking quantities to the base year. Laspeyre's price index compares the theoretical cost in a given year and the actual cost in the base year of maintaining a standard of living as in the base year.
Block V: Advanced Statistics 6 Laspeyre's price index can
be calculated using the following formula: 100 x Q P QP 0 0 0 1 ? ?
Where, P 1 = Prices in the current year P 0 = Prices in the base year Q 0 = Quantities in the base year. Laspeyre's quantity index
can be calculated by using the formula: 1 0 0 0 Q
P x100 Q P ? ?
Where, Q 1 = Quantities in the current year and Q 0 , P 0 are as defined earlier. Advantages of Laspeyre's Index Laspeyre's quantity index can be compared for all years without determining the price for each year. Disadvantages of Laspeyre's Index ? Weights in the base year may soon become out dated and no longer representative. ? When the price increases normally consumption decreases. When base year quantities result in assigning higher weight to price that has increased, it leads to higher Laspeyre's index. ? Similarly, when the prices go down, consumers tend to demand more of those items whose prices have declined the most and hence the usage of base period quantities will result in assigning lower weights to prices that have decreased the most and the net result is that the numerator of the Laspeyre's index will again be too large. PAASCHE'S METHOD Paasche's index uses the quantity measures for the current period rather than for the base period. It can be calculated using the following formula. Paasche Price Index = 100 x Q P QP 1 0 1 1 ? ? Where, P 1 = Prices in the current year P 0 = Prices in the base year Q 1 = Quantities in the current year.
Paasche Quantity Index = 1 1 0 1 Q P 100 Q P ? ? ?
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Where, P 1 = Prices in the current year Q 1 = Quantities in the current year Q 0 = Quantities in the base year. Disadvantages of Paasche Method: When new weights arrive, the entire past series must be revised. Comparison between Laspeyre's Index and Paasche's Index i. Laspeyre's index has advantages over Paasche's index, as in the latter weights are to be determined every time while considering an index number. ii. Laspeyre's index has an upward bias while Paasche's index has a downward bias. iii. Laspeyre's and Paasche's methods tend to produce opposite extremes in index values computed from the same data. iv. Under a normal situation, when consumption has not changed drastically between base period and current year, both index numbers can give satisfactory result. v. When there is equal variation in price/quantities of all items in the index, the two indices will give same result. Fixed-Weight Aggregates Method It uses neither the base-period (Laspeyres), nor most recent period prior to the base month (Paasche), but uses a weight from some intermediate period – possibly an average weighting of several periods. The weights are normally fixed and are not affected by selection of the base period. Steps in Calculating the Fixed-Weight Aggregates Method ? Multiply the current-period price by the fixed weight and add the results. ? Multiply the base-period price by the fixed weight and add the results. ? Fixed-weight aggregate Price index = n 0 n 1 QP QP Where, P 1 = Price in the current year P 0 = Price in the base year Q n = Quantities in the fixed year.

Fisher's Ideal Method Fisher's ideal method is the geometric mean of the Laspeyre's and Paasche's indices. It is given by the following formula: Fisher's Ideal Price Index = 1 0 1 1 0 0 0 1
Q P QP x Q P QP ? ? ? ? x 100 Fisher's Ideal Quantity Index = 1 0 1 1 0 0 0 1 P Q P Q x P Q P Q ? ? ? ? x 100
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Advantages of Fisher's Ideal Index i. It is based on the geometric mean, which is supposed to be the average for the construction of index numbers. ii. Fisher's ideal index is free from any upward and downward bias, since it uses both current year and base year as a weight. iii. Fisher's ideal index satisfies both the time reversal test and factor reversal test (explained latter). Example 1 Consider the following information: Commodity Year 0 (Base year) Year 3 Price (Rs./Kg.) Quantity (Kg.) Price (Rs./Kg.) Quantity (Kg.) Wheat 7.50 5000 10.25 4800 Rice 10.00 6000 14.00 5500 Pulses 25.00 3000 28.00 3500 Sugar 10.60 3000 14.50 2400 Salt 2.60 500 4.50 600 Calculate Fisher's ideal price index for the year3. Fisher's ideal price index = 1 0 1 1 0 0 0 1 PQ PQ x P Q P Q ? ? ? ? x 100 ?P 1 Q 0 = (10.25

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x 5,000) + (14.00 x 6,000) + (28.00 x 3,000) + (14.50 x 3,000) + (4.50 x 500) = Rs.2,65,000 ?P 1 Q 1 = (10.25 x 4,800) + (14.00 x 5,500) + (28.00 x 3,500) + (14.50 x 2,400) + (4.50 x 600) = Rs.2,61,700 ?P 0 Q 0 = (7.50 x 5,000) + (10 x 6,000) + (25 x 3,000) + (1,060 x 3,000) + (2.60 x 500) = Rs.2,05,600 ?P 0 Q 1 = (7.50 x 4,800) + (10 x 5,500) + (25 x 3,500) + (10.60 x 2,400) + (2.60 x 600) =

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x 5,000) + (14.00 x 6,000) + (28.00 x 3,000) + (14.50 x 3,000) + (4.50 x 500) = Rs.2,65,000 ?P 1 Q 1 = (10.25 x 4,800) + (14.00 x 5,500) + (28.00 x 3,500) + (14.50 x 2,400) + (4.50 x 600) = Rs.2,61,700 ?P 0 Q 0 = (7.50 x 5,000) + (10 x 6,000) + (25 x 3,000) + (1,060 x 3,000) + (2.60 x 500) = Rs.2,05,600 ?P 0 Q 1 = (7.50 x 4,800) + (10 x 5,500) + (25 x 3,500) + (10.60 x 2,400) + (2.60 x 600) =

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x 5,000) + (14.00 x 6,000) + (28.00 x 3,000) + (14.50 x 3,000) + (4.50 x 500) = Rs.2,65,000 ?P 1 Q 1 = (10.25 x 4,800) + (14.00 x 5,500) + (28.00 x 3,500) + (14.50 x 2,400) + (4.50 x 600) = Rs.2,61,700 ?P 0 Q 0 = (7.50 x 5,000) + (10 x 6,000) + (25 x 3,000) + (1,060 x 3,000) + (2.60 x 500) = Rs.2,05,600 ?P 0 Q 1 = (7.50 x 4,800) + (10 x 5,500) + (25 x 3,500) + (10.60 x 2,400) + (2.60 x 600) =

Rs.2,05,500 ? Fisher's Ideal Price Index = 2,65,000 2,61,700 x 2,05,600 2,05,500 x 100 =
Rs.128.12
Unit 13: Index Numbers 9 Marshall-Edgeworth Method Like the Fisher's ideal method, it also considers current year as well as the base year prices and quantities. Marshall-Edgeworth Index can be computed using the following formula,
Marshall-Edgeworth Index = 0 1 0 1 1 0
P)Q Q(P)Q Q(? ? ? ? x 100 Or 1 0 1 1 0 0 0 1 P Q + P Q P Q + P Q ? ? ? ?
x 100 Example 2 A dealer of grocery-items requires an overall comparison of the prices of the commodities he deals in, between the years Year0 and Year3. The following information is provided by him: Commodities Sugar Salt Wheat Rice Pulse P 0 (Q 0 + Q 1) 15750 1512.50 10500 18450 28600 P 1 (Q 0 + Q 1) 20650 2337.50 13125 22500 40950 Calculate the Marshall-Edgeworth price index for the year 3, using the year 0 as the base year. Marshall-Edgeworth price index = 0 1 0 1 1 0 P)Q Q(P)Q Q(? ? ? ? x 100 = 100 x 74,812.50 99,562.50 = 133.08 13.7.3 Value Index Numbers Value index considers both price changes and quantity changes over a period of time. It can be calculated by the following formula,
= 100 x Q P
QP 0 0 1 1 ? ?

Where, P 1 , Q 1 , P 0 and Q 0 follow the usual notations. The main benefit of using Value index is that it takes into consideration both price change and quantities change. 13.7.4 Average of Relatives Methods In average of relatives method, price/quintiles of current year are expressed as the price of base year and this average is expressed as the index number.

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Advantages i. Gives equal importance to every item in the index. ii. Satisfies the unit test. Disadvantages It does not satisfy all the criteria of an index number. Two Types of Average of Relatives Method i. Unweighted Average of Relatives Method ii. Weighted Average of Relatives Method. 13.8 Unweighted Average of Relatives Method The unweighted average of relatives price index for all items is obtained using any one of the measures of central tendency namely, arithmetic mean, geometric mean, median, mode or harmonic mean. Usually arithmetic mean is used to take the average of the price relatives. Unweighted average of relatives index = 1 0 P x100 P n ? ? ? ? ? ? ? Where, P 1 = Prices in the current/given year P 0 = Prices in the base year n = Number of products/items in the composite. The ratio P 1 /P 0 is the price relative. In quantity index, the quantity relative is to be calculated and then index number can be computed using any of the average methods. Unweighted average of relatives quantity index = 1 0 Q x100 Q n ? ? ? ? ? ? ? Where, Q 1 = Quantities in the given period Q 0 = Quantities in the base year n = Number of elements in the composite. Limitations ? Assigning of equal weightages is undesirable because some relatives are economically more significant than others. ? Higher price commodities receive greater weight than lower price commodities. It means it depends on the magnitude of price. ? It can mislead if appropriate average is not selected.

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Weighted Average of Relatives Method The weighted average of relatives index can be expressed mathematically as: Weighted average of relatives price index = n n n n 0 1 Q P)Q (P 100 x P P ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Where, P 1 = Prices in the current year P 0 = Prices in the base year P n = Prices in the fixed period (could also be base or current period) Q n = Quantities in the fixed period (could also be base or current periods)

P n Q n =

Value in the fixed period (can be replaced by P 0 Q 0 or P 1 Q 1 as the case may be). Some Facts: i. If the weighted average of relatives price index is to be calculated with the base values, it is the same as Laspeyres' method. Weighted Average of Relatives Price Index = 0 0 0 0 0 1

Q P)Q (P 100 x P P ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? = 0 0 0 0 0 1 Q P 100 x Q Px P P ? ? = 100 x Q P QP 0 0 0 1 ? ?

ii. If the weighted average of relatives price index is to be calculated with the current year quantity and base year price, it is the same as Paasche's method. Weighted Average of Relatives Price Index = 1 0 1 0 0 1

Q P)Q (P 100 x P P ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? = 0 0 1 0 0 1 Q P 100 x Q Px P P ? ? = 100 x Q P QP 0 0 1 1 ? ?

Example 3 Consider the following information for the year 0: Calculate the weighted average of relatives price index for the year.

Block V: Advanced Statistics 12 Weighted average of relatives price index = 1

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n n 0 n n P 100 P Q P P Q ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? Commodity 100 P P 0 1 ? P n Q n n n 0 1 Q P 100 P P ? ? ? ? ? ? ? ? ?

Rice 118.42 10,000 11,84,200 Wheat 130.77 4,050 5,29,618.50 Salt 160.00 1,300 2,08,000 Sugar 145.00 9,600 13,92,000 Pulses 137.78 13,250 18,25,585 Total 38,200 51,39,403.50 1 n n 0 P 100 P Q P ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? ? = 51, 39,403.50 ? P n Q n = 38,200.

Weighted average of relatives price index for year 0 = 51,39,403.50 134.54 38,200 ? . Check Your Progress - 1 1. When the base year values are used as weights, the weighted average of relatives price index is the same as a. The Laspeyres price index b. The Paasche's price index c. The unweighted average of relatives price index d. Fisher's ideal index e. Marshall-Edgeworth index. 2. Which of the following is/are true? a. Laspeyres Index tends to overestimate the rise in prices or has an upward bias. b. As opposed to the Laspeyres Index the Paasche Index has a downward bias. c. Fisher's Ideal Index is free from any bias. d. Both (a) and (b) above. e. All of (a), (b) and (c) above.

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Consider the following data Commodities Year 0 Year 1 Price Quantity Price Quantity A 4 16 16 13 B 7 10 17 14 C 7 25 10 32 D 3 18 6 22 What is the Laspeyres, Paasche's price indices for Year 1? Use Year 0 as the base year. a. 144.3526 and 142.7273 b. 142.5623 and 140.3025 c. 143.25 and 141.2362 d. 145.23 and 143.2562 e. 146.2325 and 141.2365. 4. The following data is available for a basket of commodities: Item Price Average quantity consumed by a family in Year 0 Year 1 Year 0 Year 1 Bread (Nos.) 6.00 8.00 360 500 Milk (liters) 10.00 15.00 540 600 Eggs (dozen) 12.00 18.00 240 300 Mutton (kg.) 80.00 120.00 100 80 What is the Fisher's ideal price index from the above data? a. 145.25. b. 147.71. c. 147.05. d. 148.32. e. 148.95. 5. What is the Marshall-Edgeworth's Index for the following data? 1 0
p q ? = 825, 1 1 p q ? = 875, 0 0 p q ? = 780, 1 0 p q ? = 800 a. 105. b. 108.60. c. 107.60. d. 109.25. e. 110.65.
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Chain Index Number One main problem in the construction of any index number is choosing an appropriate base period. Usually statisticians want to use a base year where prices (or volumes) are not unnaturally high or low. Sometimes it is better to use chain-based system where, in calculating successive index numbers, the base used is the previous period. A chain-based index number is particularly suited for period by period comparisons, but a fixed-base index number makes it easier to compare the movement of prices over time. Chain index for a given year Average link relative
of the given year x Chain index of previous year = 100 Where, Link relative = Price in a given period , The previous year s price
x 100 The following steps are to be noted in the construction of a chain index: i. Calculate link relatives. ii. Obtain the average link relative of the year. This is the ratio of the total link relatives for each year to the number of items in the index. iii. Successive multiplication of average link relatives to be done. Advantages of Chain Base Method ? The key advantage of this method over the fixed base year technique is that it uses prices from a period to help compute real growth for that period. ? The link relatives calculated by using the chain base method enable comparisons over successive years. ? Chain base method enables the introduction of new items and the deletion of old items without altering the original series. It is thus flexible. ? Whenever found necessary, weights can be adjusted in the chain base method. ? Seasonal variations have minimal impact on chain index numbers. Disadvantage of Chain Base Method ? As percentages are chained together long range comparisons are not very accurate. ? The chain-weighted procedure could add considerable complexity to the construction of index number.
Unit 13:
Index Numbers 15 13.11 Tests for Consistency In this section, we will discuss the mathematical criteria for judging the appropriateness of a particular index number. Some important tests are: a. Time reversal test b. Factor reversal test c. Circular test. 13.11.1 Time Reversal Test For deciding the appropriations of particular formula of index number Prof. Irvin suggested the time reversal test. According to this test, when the data for any two years are treated by the same method but with bases reversed, the product of two obtained index numbers should be unity. Mathematically, we can say that P 0, 1 x P 1, 0 = 1 Where, P 0, 1 = Original index for the current year with a given base year. P 1, 0 = Resulting index, with time periods reversed, for the base year with the current year as a base year. Table 3: Application of Time Reversal Test Methods which Satisfy the Test Methods which Do Not Satisfy the Test Fisher's Ideal Index, Marshall-Edgeworth Index. Laspeyre's Price Index, Paasche's Index. 13.11.2 Factor Reversal Test According Prof. Irving Fisher the product of price index and quantity index should be equal to value index for the year. In other words, this method permits interchanging of the prices and quantities without giving any inconsistent result. Suppose, P 1 = Prices in the current year P 0 = Prices in the base year Q 1 = Quantities in the current year Q 0 = Quantities in the base year. Then, P 0, 1 x P 1, 0 = 0 0 1 1 Q P QP ? ? (Value ratio).
Block V: Advanced Statistics 16 13.11.3 Circular Test Circular test is nothing but an extension of the time reversal test. This test can be mathematically expressed as P 0,1 x P 1,2 x P 2,0 = 1 Table 4: Application of Circular Test Methods which Satisfy the Test Methods which Do Not Satisfy the Test Fixed Weights Aggregate Index and Simple Aggregates Method. Laspeyre's Price Index, Paasche's Index. Check Your Progress - 2 6. Which of the following statement is true as per Prof. Irving Fisher? a. Price index x Quantity index = Value index b. Price index x Value index = Quantity index c. Value index x Quantity index = Price index d. Price index + Quantity index = Value index e. Value index – Quantity index = Price index 7. Which of the following tests is a test of consistency? a. Run reversal test b. Factor reversal test c. Circular reversal test. d. Run Test e. F test 8. What is the product result of original index and Resulting index? a. 0 b. 1 c. 2 d. 3 e. 4 9. Which of the following is used to calculate Link relative? a. Quantity b. Value c. Price d. Weighted Price e. Weighted Quantity
Unit 13: Index Numbers 17 10.

Which of the following is the advantage of chain based index number? a. The link relatives calculated by using the chain base method enable movement prices. b. Chain base method enables the introduction of new items and the deletion of old items without altering the original series. It is thus flexible. c. Weights are fixed in the chain base method. d. Seasonal variations have enough impact on chain index numbers. e. As percentages are chained together long range comparisons are very accurate. 13.12 Summary ? Index number is a statistical measure designed to show changes in a variable or a group or related variables with respect to time, geographic location or other characteristics such as income, profession, etc. ?
Index number is calculated as a ratio of the current value to a base value
and expressed as a percentage. Price index, Quantity index and Value index are three major Index Numbers. ? There are four methods to weight an index; they are – Paasche's method, Laspeyre's method, Fisher method and Fixed-weight aggregate methods. 13.13 Glossary Fixed-Weight Aggregates Method: To weigh an aggregates index, this method uses quantities consumed during some representative period as weights. Index Number: A statistical measure in the form of ratio that measures how much a variable changes over time. Laspeyre's Method: To weight an aggregates index, the Laspeyers' method uses the quantities consumed during the base period as weights. Paasche's Method: In weighting an aggregates index, the Paasche method uses the quantities consumed during the current period as weights. Price Index: Compares levels of prices from one period to another. Quantity Index: A measure of how much the number or quantity of a variable changes over time. Weighted Aggregates Index: Using all the values considered, this index assigns weights to these values. Weighted Average of Relatives Method: To construct an index number, this method weighs by importance the value of each element in the composite.
Block V: Advanced Statistics 18 13.14 Suggested Readings/Reference Material 1.
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Self-Assessment Questions 1. How Index numbers help government and other various private organization? 2. Explain the difference between the Laspeyre's Method and Paasche Method. 3. Prove that the weighted average of relatives price index calculated with the base values, is same as Laspeyre's method. 4. Factor reversal test is satisfied only by Fisher's ideal Index number. Justify. 5. What is circular test? Show that Laspeyre's Price Index and Paasche Index do not satisfy the circular test. 13.16 Answers to Check Your Progress 1. (a) The Laspeyres price index 2. (e). Laspeyres Index tends to overestimate the rise in prices or has an upward bias. As opposed to the Laspeyres Index the Paasche Index has a downward bias. Fisher's Ideal Index is free from any bias. 3. (a) 144.3526 and 142.7273 4. (b) 147.71
Unit 13: Index Numbers 19 5. (
c) 107.60 6. (a) Price index x Quantity index = Value index 7. (b) Factor reversal test 8. (b) 1 9. (c) Price 10. (b) Chain base method enables the introduction of new items and the deletion of old items without altering the original series. It is thus flexible.
Unit 14 Simulation Structure 14.1 Introduction 14.2 Objectives 14.3 Analytical Approach 14.4 Simulation 14.5 Summary 14.6 Glossary 14.7 Suggested Readings/Reference Material 14.8 Self-Assessment Questions 14.9 Answers to Check Your Progress 14.1 Introduction In the previous unit
we have
learnt

the calculation of different index numbers. The techniques of decision-making process have already been discussed in the unit Probability Distribution. This unit aims to choose an action from several available alternatives which would, in some sense, optimize the results obtained. The process of selection of the optimal alternative can be subjective or quantitative. Subjective approach decision is based on the previous experience but subjective methods involve some mathematical models. This unit provides an insight into: Analytical approach, Simulation. 14.2
Objectives After going through the unit, you should be able to: ?
Demonstrate Analytical approach to decision-making; and ? Explain Simulation approach to decision-making. 14.3 Analytical Approach Analytical approach is a simple technique used in decision-making. For example, a vendor selling mangoes everyday wants to fix their price on a particular day. It could be equal to the competitor's price or above or below it. He has the following information about the historical sales. Price (Rs.) Quantity (Kg.) 16 60 15 80 17 40
Unit 14: Index Numbers 21 The purchase cost of mangoes is Rs.10 per kg and he has to pay Rs.2 per kg as municipal tax. If he purchases more than 50 kg he can get a discount of Re.1 per kg and if he purchase 75 kg of mangoes he can get a Rs.2 discount. To find out the price that the vendor should charge, we must first determine his objective. Let us assume that his objective is to maximize profits. To find out the price which would maximize the profits, we construct the following table: Price (Rs.) Sales Quantity (Kg.) Sales Value Cost (Rs.) Profit (Rs.) 16 60 960 660 300 15 80 1,200 800 400 17 40 680 480 200 We thus find that the profits are maximized at the price of Rs.15 per unit, and therefore this price should be chosen. Analytical approach is easy to use but it is not possible to use everywhere, specifically when there is uncertainty about variables. 14.4 Simulation Simulation technique is used when it is inconvenient, dangerous, expensive or impossible to do experiment with the actual system, or even impossible to builds a physical model. In other words, when decisions are to be taken under conditions of uncertainty, simulation can be used. An example of simulation is testing of an aircraft in a wind tunnel. Based on this experiment, performance of an aircraft is determined for real operating conditions. Testing a vehicle on the test track is also a good example of simulation. Exhibit 14.1 shows how L'Oreal uses simulation for recruitment. Exhibit 14.1: Application of Simulation in L'Oreal L'Oreal uses online simulation game 'reveal the game' ? Applicants challenge themselves in the development of a new product launch using a real-world scenario. ? Applicants can compete with each other and share results. The company determined the best fit within the company for an individual based on his/her strengths. Adapted from https://www.gamification.co/.../loreal-uses-serious-games-employee-recruitment/ Simulation has wide use in statistical as well as physical models. A statistical model takes random samples from some probability distribution that describes the operation of some aspect of a system, or the total system. Monte-Carlo simulation technique takes random samples and is one of the widely used techniques in statistics.
Block V: Advanced Statistics 22 Some Important Terms Let us discuss some basic terms used in simulation: i. System: It is the segment that is to be studied or understood to draw conclusions. ii. Decision Variables: They are used under differing sets of circumstances and determined through the process of simulation. iii. Environmental Variables: They describe the environment and are dependent upon that environment in which the system operates. iv. Endogenous Variables: They are generated within the system itself. v. Criterion Function: Endogenous variables are used as the criterion function for evaluating the performance of the system. Sometimes more than two endogenous variables can be used as criterion function. Example 1 A company is planning a new venture and wants to use simulation to analyze the risk of the project. It has identified annual demand, sales price and variable costs, as the factors on which the simulation is to be carried. Identify the decision variables, environmental variables, endogenous variables, and the criterion function. Decision Variables Acceptable risk of the project Environmental Variables Risk in the same type of project or average risk Endogenous Variables Quantity sold, sales revenue, total cost and profit Criterion Function Maximize profit 14.4.1 Constructing a Model Mathematical modeling requires the setting up of mathematical relationships which would represent the system. Let us explain with an example: Suppose for the previous example (of mangoes) we want to know the return on investment. Let us assume that the competitors' average price is P c and the price charged by the vendor is P. If quantity sold is Q and initial investment of the project is I, then, we can express this relationship mathematically as: Q = f(P, P c) If we assume the total cost of quantity sold as C, then C = g(Q) The total cost is a function of quantity sold. If π is the profit earned by the vendor, then Profit = π = PQ – C = P f(P, P c) – g(Q) Return on Investment (I) = ExpectedTotalProfit InitialInvestment IntialInvestment ? The above equation represents a mathematical model of the system.

Unit 14: Index Numbers 23 14.4.2 Need for Simulation Even though it is easy to set-up, an analytical model, it has some drawbacks: a. Sometimes, it may be difficult or impossible to perform experiment when interrelationships are too complex. b. Typically, predict only average or steady-state behavior. c. It may be very costly to perform a real life problem. Though there are some other alternative methods like linear programming, sensitive analysis, dynamic programming, each and every method has its own pros and cons. The best alternative of the analytical model is simulation. A typical simulation technique is based on random number generation and is easy to understand. A simulation process requires the following steps: i. Identify the problem ii. Develop the simulation model iii. Test the model iv. Develop the experiments v. Run the simulation and evaluate results. Repeat until results are satisfactory. 14.4.3 Random Number Generation Random variable is a numerical value associated with different outcomes as a result of random experiment. In the previous example, if we want to know the profits for different situations, the easiest process is to select some random value from a random table to perform the experiment. This random number can be generated using various techniques. These techniques may be manual or computerized. Check Your Progress - 1 1. One of the following is not a reason for using simulation technique. Identify. a. Inconvenient b. Dangerous c. Expensive d. For doing experiments 2. A company is planning a new retail stores to be opened and wants to use simulation to analyze the risk of the project. It has identified the total likely yearly visitors, sale prices of products and variable costs over the year, as the factors on which the simulation is to be carried. Identify the decision variables. a. Acceptable risk of the project b. Risk in the same type of project or average risk

Block V: Advanced Statistics 24 c. Quantity sold, sales revenue, total cost and profit d. Maximize profit 3. Priya Darshini hotel group company is planning five new hotel to be opened in Hyderabad and wants to use simulation to analyze the risk of the project. It has identified the total likely yearly visitors, sale prices of eatables and local variable costs over the year, as the factors on which the simulation is to be carried. Identify the environmental variables. a. Acceptable risk of the project b. Risk in the same type of project or average risk c. Quantity sold, sales revenue, total cost and profit d. Maximize profit 4. KLM textile departmental stores is planning a six retail stores to be opened in and around Hyderabad, and wants to use simulation to analyze the risk of the project. It has identified the total likely yearly visitors, sale prices of various textiles and variable costs of various textiles over the year, as the factors on which the simulation is to be carried. Identify the endogenous variables. a. Acceptable risk of the project b. Risk in the same type of project or average risk c. Quantity sold, sales revenue, total cost and profit d. Maximize profit 5. CMR jewelers is planning a four retail stores to be opened within Hyderabad city and wants to use simulation to analyze the risk of the project. It has identified the total likely yearly visitors, sale prices of various ornaments and gold products and variable costs of gold and silver metals over the year, as the factors on which the simulation is to be carried. Identify the criterion function a. Acceptable risk of the project b. Risk in the same type of project or average risk c. Quantity sold, sales revenue, total cost and profit d. Maximize profit 6. A simulation process follows a defined set of steps: Identify the problem, Develop the simulation model, Develop the experiments, Run the simulation and evaluate results. Identify the missing step. a. Test the model b. Test the experiment c. Design the experiment d. Generate random numbers

Unit 14: Index Numbers 25 7. Team at Reliance fresh was trying to maximize the product mix of sales in a given day. They preferred to use good alternative methods for analytical approach. What is the best alternative of the analytical model? a. Linear programming b. Sensitive analysis c. Dynamic programming d. Simulation 14.4.4 Monte-Carlo Simulation Monte-Carlo simulation is a widely used method of simulation and is used when a process has a random component. The process involved is as follows: ? Identify a probability distribution ? Choose intervals of random numbers to match probability distribution ? Obtain the random numbers ? Interpret the results. 14.4.5 Advantages of Simulation Simulation technique has various advantages over other techniques, some of which are detailed hereunder: i. It is easy to understand and a comparatively superior mathematical technique. It is flexible i.e., we can modify and adjust the model according to the problems. ii. Simulation allows an experiment without interfering the real system. It can represent a real-world process more realistically because fewer restrictive assumptions are required. iii. It can perform large calculations in a few minutes. iv. It may be sometimes the only alternative to provide solutions to the problem under study. For example, it is not possible to obtain transient (time-dependent) solutions for complex queuing models in closed form or by solving a set of equations, but such solutions are readily obtained with simulation methods. v. It is cheaper than actual experimentation; in some cases, it may be the only reasonable initial approach, as when the system does not yet exist but theoretical relationships are well-known. For example, in case of the armed forces, it would be impossible to train a person when an actual war is going on. Rather, all the conditions that would prevail during a war are reconstructed and enacted so that the trainee develops the skills and instincts required of him during combat. Thus, war conditions are simulated to impart training.

Block V: Advanced Statistics 26 14.4.6 Disadvantages of Simulation Though simulation has various advantages over the other techniques it is not free from drawbacks. Some of the disadvantages are explained below: i. It does not produce optimal solutions. As the number of parameters increases, the difficulty in finding the optimum values too increases to a great extent. ii. It is computer intensive. iii. It can be used only when there is uncertainty and not always. iv. Sometimes, it can be more expensive than real life experiments. Check Your Progress - 2 8. Monte-Carlo simulation is a widely used method of simulation and is used when a process has a random component. The process involved has these steps: i. Choose intervals of random numbers to match probability distribution ii. Interpret the results iii. Obtain the random numbers, iv. Identify a probability distribution,

a. i, ii, iii, iv b. ii, iii, I, iv c. ii, iv, iii, i d. iv, i, iii,

ii 9. Though simulation has various advantages over the other techniques it is not free from drawbacks. Which of the following is not an identified disadvantage of simulation a. May not produce optimal solutions b. Difficulty in finding the optimum value increases with parameters c. Can be more expensive than real life experiments d. Cheaper than actual experimentation 10. Which one of these is not an identified advantages of simulation? a. Easy to underrated and flexible b. Doe not interfere with the real system c. Only alternative to the problem under study d. Computer intensive 14.5 Summary ? Decision-making involves choosing an action from several available alternatives. In a business, the idea is to choose the course of action which would in some sense optimize the results obtained.

Unit 14: Index Numbers 27 ? When quantitative or mathematical methods are applied, we may adopt two approaches. Analytical approach and Simulation. Simulation technique is used when decisions are to be taken under conditions of uncertainty. 14.6 Glossary Decision Variables: Variables that are determined through the process of simulation. Endogenous Variables: Variables that are generated within the system itself. Environmental Variables: They describe the environment and are dependent upon that environment in which the system is operating. Simulation: A technique used when decisions are to be taken under conditions of uncertainty. 14.7 Suggested Readings/Reference Material 1.

Gupta, S. P. Statistical Methods. 46th Revised ed. New Delhi: Sultan Chand & Sons. 2021. 2. I. Levin Richard, H. Siddiqui Masood, S. Rubin David, Rastogi Sanjay. Statistics for Management. Pearson Education; Eighth edition, 2017. 3.

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Self-Assessment Questions 1. Briefly explain the purpose of simulation in decision-making. 2. Explain the terms decision variables, environmental variables, and endogenous variables used in simulation technique. 3. What are the advantages and disadvantages of using simulation in decision- making? 4. What is Monte-Carlo simulation? Explain. 5. Explain the basic difference between the analytical approach and simulation.

Block V: Advanced Statistics 28 14.9 Answers to Check Your Progress 1. (d) For doing experiments Simulation technique is used when it is inconvenient, dangerous, expensive or impossible to do experiment with the actual system, or even impossible to build a physical model 2. (a) Acceptable risk of the project Decision Variables: They are used under differing sets of circumstances and determined through the process of simulation. 3. (b) Risk in the same type of project or average risk Environmental Variables: They describe the environment and are dependent upon that environment in which the system operates. 4. (c) Quantity sold, sales revenue, total cost and profit Endogenous Variables: They are generated within the system itself. 5. (d) Maximize profit Criterion Function: Endogenous variables are used as the criterion function for evaluating the performance of the system. Sometimes more than two endogenous variables can be used as criterion function. 6. (a) Test the model A simulation process requires the following steps: i. Identify the problem, ii. Develop the simulation model, iii. Test the model, iv. Develop the experiments, v. Run the simulation and evaluate results 7. (d) Simulation Though there are some other alternative methods like linear programming, sensitive analysis, dynamic programming, each and every method has its own pros and cons. The best alternative of the analytical model is simulation 8. (d). iv, i, iii, ii Monte-Carlo simulation is a widely used method of simulation and is used when a process has a random component. The process involved is as: Identify a probability distribution, Choose intervals of random numbers to match probability distribution, Obtain the random numbers, Interpret the results. 9. (d). Cheaper than actual experimentation Some of the disadvantages identified are: i. It does not produce optimal solutions. ii. As the number of parameters increases, the difficulty in finding the optimum values too increases to a great extent. iii. It is computer intensive.

Unit 14: Index Numbers 29 iv. It can be used only when there is uncertainty and not always. v. Sometimes, it can be more expensive than real life experiments. 10. (d) Computer intensive Identified advantages are: i. It is easy to understand, and flexible ii. Simulation allows an experiment without interfering the real system. iii. It can perform large calculations in a few minutes. iv. It may be sometimes the only alternative to provide solutions to the problem under study v. It is cheaper than actual experimentation

Unit 15 Linear Programming Structure 15.1 Introduction 15.2 Objectives 15.3 Overview

of Linear Programming 15.4 Formulation of Linear Programming Problems 15.5 The Graphical Method 15.6 The Simplex Method 15.7 Post-Optimal Analysis 15.8 Duality 15.9 The Transportation Problem 15.10

Summary 15.11 Glossary 15.12 Suggested Reading/Reference Material 15.13 Self-Assessment Questions 15.14 Answers to Check Your Progress Questions 15.1 Introduction In the previous unit,

we

learnt simulation and its uses. This unit, expounding the basis of linear programming is divided into four sections. Section I presents the definition and application of linear programming. Section II presents the graphical method of solving the linear programming problem. Though this method can be applied only to those problems having only two basic variables, it is a very useful pedagogic device to understand certain concepts underlying the more advanced methods of linear programming. Section III discusses the simplex method which uses a very efficient algorithm to determine the optimal solution to a linear programming problem. The computation rules and procedures associated with the simplex method are so well defined that electronic computers can profitably use them to easily obtain solutions. Section IV touches on some elements of post-optimal analysis and Transportation problem. 15.2

Objectives By the end of this unit, students should be able to: ? Explain and use linear programming in industry context. ? Compute linear programming problems. ? Determine the solution of linear programming problems. ? Assess the transportation problem in linear programming.

Unit 14: Index Numbers 31 15.3 Overview

of Linear Programming Linear programming, one of the important techniques of operations research,

has been applied to a wide range of business problems. This technique is useful in solving decision making problems which involve maximizing a linear objective function subject to a set of linear constraints. Linear programming is helpful in solving a variety of problems in finance, budgeting and investments. The important applications of this technique are in the following areas: ? Selection of a product mix which maximizes the profits of the firm subject to several production, material, marketing, personnel and financial constraints. ? Determination of the capital budget which maximizes the net present value of the firm subject to several financial, managerial, environmental, and other constraints. ? Choice of mixing short-term financing which minimizes the cost subject to certain funding constraints. For instance,
Airlines use linear programming to optimize their profits according to different seat prices and customer demand. Airlines also use linear programming for pilot scheduling and routes. Optimization via linear programming increases airlines' efficiency and decreases expenses. 15.3.1
Review of Linear Functions The function of a variable has been defined earlier in the chapter 'Functions and Calculus'. Linearity represents a special case of the relationship y = f(x). The relationship is defined as linear if, for all possible values of x and y, a given change in the value of x brings about a constant change in the value of y. Example 1 Consider the following function, y = 4 + 3x The values of x and y and the changes in their values are given below: x Change in x y Change in y −5 – −11 – −4 1 −8 3 −3 1 −5 3 −2 1 −2 3 −1 1 1 3 0 1 4 3 1 1 7 3 2 1 10 3 3 1 13 3 A plot of values of x and the corresponding values of y will trace a straight line.
Block V: Advanced Statistics 32 The general expression of a linear function of one independent variable is: y = a + bx where, x = independent variable y = dependent variable a = a numerical constant called 'intercept' b = a numerical constant called 'slope' The expression for a linear function of n independent variables,

44%	MATCHING BLOCK 4/64	W
x 1 , x 2 , x 3 , ...x n , is y = a 0 + a 1 x 1 + a 2 x 2 + a 3 x 3 +.......+ a n x n where a 0 , a 1 , a 2 , a 3 , ... a n		

are given numerical constants. Some Important Results The technique of solving linear programming problems is based on the study of the properties of linear functions.
Let us consider a linear function of one independent variable, y = a + bx, which is shown in the graph below. Figure 15.1 In linear programming, we come across expressions of the following type: y – bx > a We will be interested in the values of x and y for which the left-hand-side value is less than, or equal to, the right-hand-side value 'a'. A line divides the plane into two parts. In one of the parts, the value of y – bx for all points in the part will certainly have a value less than the right-hand-side value 'a', and in the other part, the value of y – bx will be definitely greater than the value of 'a' for all the points in the part, whereas for points on the line, the value of y – bx will be exactly equal to 'a'. Example 2 Consider the equation: y – 3x = 4
Unit 14: Index Numbers 33 This line is graphically represented below. Figure 15.2 For all points on the line AB, the co-ordinates satisfy the equation: y – 3x = 4 For any point below the line, i.e. Part I, we will have: y – 3x > 4 and for points above the line, i.e. Part II, we will have:
y – 3x < 4 This can be easily verified by taking some points below and above the line. In case of n number of variables x 1 , x 2 , x 3 , ...,x n , we get a graph in n- dimensional space with n co-ordinates. For any numerical values such as a 1 , a 2 , a 3 ....,a n and b, we define the equation for a plane as:
a 1 x 1 + a 2 x 2 +......+ a n x n = b This
plane divides the n-dimensional space into two parts. In one part, the co-ordinates of the points satisfy the inequality a 1 x 1 + a 2 x 2 +......+ a n x n > b and in the other part, the co-ordinates of the points satisfy the inequality a 1 x 1 + a 2 x 2 +......+ a n x n < b. 15.4
Formulation of Linear Programming Problems Formulating a linear programming problem is the most vital and difficult aspect of solving a real problem. Though there is no fixed pattern for formulating such problems, the following procedure can be followed: 1. Identify the Decision Variables – The decision-maker should identify the variables that are under his/her control. These variables, which can be changed in order to optimize the objective function, are called decision variables and they should be defined completely and precisely.
Block V: Advanced Statistics 34 2.
Define

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the Objective Function – The objective of the problem and the criteria for evaluating alternative solutions should be well defined. The objective is generally written as a linear function of the decision variables, each multiplied by an appropriate coefficient. 3. Identify and Express Relevant Constraints – After defining the decision variables and the objective function, the operations manager should identify the constraints that affect the objective function. This process of formulation is generally iterative.
Refer
Exhibit 15.1 for an example demonstrating how an operations manager can determine whether the linear programming technique is applicable to a particular problem. After ensuring that the linear programming can be applied to the problem, the next step is to formulate the problem. Exhibit 15.1: Recognizing Linear Programming Problem As a part of its strategic planning process, the Gulf Coast Company must determine the mix of its products to be manufactured next year. The company produces two principal product lines for the commercial construction industry: a line of powerful portable circular saws and a line of precision table saws. The product lines share the same production capacity and are sold through the same sales channels. Although some product variety does exist within each product line, the average profit is Rs. 5 for each circular saw and Rs. 7 for each table saw. The production capacity is constrained by the capacities of two facilities: fabrication and assembly. A maximum of 13 hours of fabrication capacity is available per month. Each circular saw requires 2 hours and each table saw requires 3 hours of fabrication respectively. There is a maximum of 12 hours of assembly capacity available per month. Each circular saw requires 3 hours and each table saw requires 2 hours of assembling respectively. How many circular saws and table saws should be produced monthly next year to maximize profit?
The general form of a linear programming problem is Maximize
n n 2 2 1 1 xC ... xC xC Z ? ? ? ?
Subject

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to the constraints A 11 x 1 +A 12 x 2 + ... +A 1n x n ≤ b 1 A 21 x 2 +A 22 x 2 +...+A 2n x n ≤ b 2 A m1 x 1 +A m2 x 2 +...A mn x n ≤ b m x 1 , x 2 , ...x n ? 0.

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A 11 x 1 +A 12 x 2 + ... +A 1n x n ≤ b 1 A 21 x 2 +A 22 x 2 +...+A 2n x n ≤ b 2 A m1 x 1 +A m2 x 2 +...A mn x n ≤ b

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A 11 x 1 +A 12 x 2 + ... +A 1n x n ≤ b 1 A 21 x 2 +A 22 x 2 +...+A 2n x n ≤ b 2 A m1 x 1 +A m2 x 2 +...A mn x n ≤ b

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A 11 x 1 +A 12 x 2 + ... +A 1n x n ≤ b 1 A 21 x 2 +A 22 x 2 +...+A 2n x n ≤ b 2 A m1 x 1 +A m2 x 2 +...A mn x n ≤ b m x 1 , x 2 , ...x n ? 0.

Where x 1 , x 2 ,
x 3 , ...x n =
a set of variables whose numerical values are to be determined
C ij , A ij ,
and b i =
numeric coefficients that are specified in the problem. It can be observed that Z is a linear function of variables x i , i.e, when the value of a variable
x i

increases by unity, the value of Z increases by C_i.
Unit 14: Index Numbers 35 The linear programming model can also be used to minimize the objective function. In such case, the constraints are written with a sign '?'. The constraints can also be written as linear equalities. Thus, the resulting set of decision variables (values for the n variables, x_1, x_2, x_3 ...x_n) optimizes (either maximizes or minimizes) the objective function, subject to 'm' constraints and the non-negativity conditions of
x_j
variables. Refer Exhibit 15.2
for the steps involved in formulating the linear programming model for the problem given.
Exhibit 15.2:
Formulating a Linear Programming Problem The problem illustrated in Exhibit 15.1 can be formulated as a linear programming problem by adopting the following steps: Step 1: Identify the decision variables - The variables that can be altered to optimize the profit of the Gulf Coast Company are the number of circular saws and table saws that are to be manufactured. Let x_1 and x_2 represent the number of circular saws and table saws manufactured per month respectively. Step 2: Define the objective function - The objective of the problem is to maximize profits. Each circular saw contributes Rs. 5 and each table saw contributes Rs. 7 toward profits. Hence, the objective function may be defined as; Maximize $Z = 5x_1 + 7x_2$ Step 3: Identify the relevant constraints: The goal of maximizing profit is constrained by the number of fabrication hours, and the number of assembly hours. These constraints can be expressed as; $2x_1 + 3x_2 \leq 13$ (Each circular saw requires 2 hours of fabrication and each table saw requires 3 hours of fabrication, but the total fabrication hours available are only 13). Similarly, $3x_1 + 2x_2 \leq 12$. (Each circular saw requires 3 hours for assembling and each table saw requires 2 hours for assembling. But the total assembly hours available are only). The other constraint is a non-negativity constraint. Since a negative number of saws cannot be manufactured,
x_1 and $x_2 \geq 0$. Thus, the
linear programming problem is finally formulated

96%	MATCHING BLOCK 10/64	W
as: Maximize $Z = 5x_1 + 7x_2$ Subject to $2x_1 + 3x_2 \leq 13$ $3x_1 + 2x_2 \leq 12$ $x_1, x_2 \geq 0$.		

100%	MATCHING BLOCK 9/64	W
Maximize $Z = 5x_1 + 7x_2$ Subject to $2x_1 + 3x_2 \leq 13$ $3x_1 + 2x_2 \leq 12$ $x_1, x_2 \geq 0$.		

Block V: Advanced Statistics 36 15.5 The Graphical Method The graphical method is a simple one, and is the most easily understood of the several linear programming methods. A thorough knowledge of the graphical procedure provides necessary insight and confidence to understand the more advanced methods and concepts behind these methods. However, it should be pointed out that the graphical method can be applied only in the case of two variables. It cannot be applied to problems with many variables. 15.5.1 Maximization Example 3 Consider a small foundry which specializes in the production of iron castings. For the sake of simplicity, assume that the foundry specializes in producing two types of castings — casting A and casting B. Because of a strong consumer demand for these products, it is assumed that the foundry can sell as many units as it produces. The profit is Rs.70 and Rs.40 for each of casting A and casting B respectively. The foundry manager should decide the quantity of these castings to be produced each week so as to maximize the total profit. Production of castings requires certain resources like raw materials, labor and foundry capacity. The requirements and their availabilities are given in the following table: Resources Required per unit of Available in a week Casting A Casting B Raw material -1 2 kgs. 1 kg. 120 kgs. Raw material -2 0.8 kgs. none 40 kgs. Labor 3 man-days 2 man-days 200 man-days Foundry capacity 4 units 3 units 360 units Let us formulate this problem in terms of mathematical equations or inequalities. As the manager has to decide the number of type A and type B castings to be produced, let us define the variables: q 1 = number of type A castings to be produced q 2 = number of type B castings to be produced For this production schedule, the total profit will be 70q 1 + 40q 2 This function is known as the objective function which is to be maximized. If there are no constraints, the profit can be increased to infinity. In real life, there are restrictions of different kinds. These are formulated as constraints. Let us consider raw material-1, of which only 120 kgs are available. If q 1 of type A castings and q 2 of type B castings are produced, then the requirement of
Unit 14: Index Numbers 37
raw material-1 is 2q 1 + 1q 2 , and this should be less than or equal to the available quantity of raw material-1. This can be shown by the following in equation: 2q 1 + 1q 2 > 120 This implies that we are interested in the values of q 1 and q 2 for which the left- hand-side value is less than or equal to the right-hand-side value of 120. Otherwise, the requirement will exceed the availability and the production of that quantity will not be feasible. By a similar argument, we get the constraints for raw material-2, labor and foundry capacity as: Raw material-2 : 0.8q 1 + 0q 2 > 40 Labor : 3q 1 + 2q 2 > 200 Foundry capacity : 4q 1 + 3q 2 > 360 As one cannot produce negative quantities, we have the restrictions: q 1 < 0, q 2 < 0 Putting together the above elements, the problem may be represented as: Maximize Z : 70q 1 + 40q 2 ...(1) Subject to constraints: 2q 1 + q 2 > 120 0.8q 1 + 0q 2 > 40 ....(2) 3q 1 + 2q 2 > 200 4q 1 + 3q 2 > 360 q 1 < 0, q 2 < 0 ....(3) We have to find the values of q 1 and q 2 which will satisfy constraints (2) and (3) and at the same time maximize function (1). The function given in (1) is called an objective function. The inequalities in (2) are called constraints and the inequalities in (3) are called non-negativity restrictions or constraints. This problem cannot be solved by the calculus method because of the inequality constraints. The first step in the graphical method of solution is to identify the region in the graph which corresponds to all pairs of values of q 1 and q 2 for which (2) and (3) are valid. Let us consider the non-negativity restrictions given by (3). The values of q 1 and q 2 which satisfy these restrictions should fall in the first quadrant of the graph. Hence, We can ignore pairs of values of q 1 and q 2 which fall in other quadrants. This is indicated by arrow marks on the q 1 -axis (or x-axis) and q 2 -axis (or y- axis) in the graph shown below.
Block V: Advanced Statistics 38 Figure 15.3
Let us next find the region corresponding to the values of q 1 and q 2 for which the first constraint 2q 1 + q 2 ??120 is satisfied. To do this, we have to first draw the line 2q 1 + q 2 = 120 For this, we need to fix two points on this line. The points that we have chosen are: q 1 = 0 q 2 = 120 and q 1 = 60 q 2 = 0 By joining these points, we get the line, and the points below the line, indicated by arrows, will satisfy the first constraint. Other constraint equations are also drawn on the graph. The region common to all the regions identified gives the set of points for which the values of the co-ordinates q 1 and q 2 satisfy constraints (2) and (3). The region identified is OABCD is called the feasible region. It may be noted that all values of q 1 and q 2 which satisfy constraints (2) and (3), lie within the region OABCD and all points in the region OABCD will have the co-ordinates q 1 and q 2 , which will satisfy constraints (2) and (3). Hence, an optimal solution to the problem should have co-ordinates q 1 and q 2 within or on the boundary of region OABCD. Now let us search for the optimal solution. Suppose, we are interested in finding a product mix which will give a profit of say, an arbitrarily chosen value of Rs.2,800. To get the product mix, we have to search in the region OABCD to examine whether any point gives a profit of Rs.2,800. The easiest way is to draw the straight line whose equation is 70q 1 + 40q 2 = 2,800
Unit 14: Index Numbers 39

and examine whether it passes through any points in the region OABCD. In the graph given below, the feasible region OABCD and the above mentioned straight line are shown. We can observe that there are many points on this straight line which come under the feasible region, and each point will give the co-ordinates which refer to the production levels that yield the same profit of Rs.2,800. For example, take the two points (40,0) and (0,70) on this line. The production levels corresponding to these points are (i) 40 of type A castings and 0 of type B castings and (ii) 0 of A castings and 70 of type B castings. It can be verified that each co-ordinate gives the same profit.
Thus, the straight line drawn is also the profit line. Figure 15.4 C 60
Suppose we wish to increase the profit, we look for a product mix which will give a profit of, say, Rs.3,080. As done earlier, we draw the line $70q_1 + 40q_2 = 3,080$ and examine whether it passes through the region OABC. This line is parallel to the first line and passes through the feasible region, thus indicating that it is possible to increase the profit to Rs.3,080. This suggests that, as we move up this line in the Northeastern direction, parallel to itself, we can obtain product mixes which will give higher and higher profits. We should move the line as far as possible without removing it completely from the region of feasible solutions as otherwise we will not find any feasible product mix which will satisfy the constraints. The optimal solution is then given by the point of final contact, which will be one of the corner points. In this case, the point is B, whose co- ordinates are (40, 40), indicating that the production level should be 40 for each of type A and B castings and this will yield a profit of 70 x 40 + 40 x 40 = 2,800 + 1,600 = Rs.4,400
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Remark The following should be now noted: a. In some cases, the corner point may have co-ordinates which may be fractional. In cases where integer solutions are required (that is, some or all variables are required to take only integer values), other techniques called integer programming methods should be used. Otherwise, the solution which gives the fractional values, may be rounded off to arrive at a solution which is an approximation of the optimal solution. b. If the objective function is parallel to an edge of the feasible region, then we get a number of product mixes, each of which will give the same maximum profit. This is a case of multiple optimal solutions. In the previous problem, consider the objective function to be $60q_1 + 40q_2$ instead of $70q_1 + 40q_2$. You may then ascertain that all the points on AB are optimal. Since the slope of the objective function is equal to that of the line $3q_1 + 2q_2 = 200$. c. The optimal solution, if it exists, should occur at one of the corner points. These corner points are also called extreme points or vertices. Thus, to find the optimal solution to a linear programming problem, we need to search only the extreme points. It can be shown that there will be only a finite number of extreme points in any given problem. 15.5.2 Minimization Example 4 A farmer is advised to utilize at least 900 kg of mineral A and 1200 kg of mineral B to increase the productivity of crops in his fields. Two fertilizers, F_1 and F_2 are available at a cost of Rs.60 and Rs.80 per bag. If one bag of F_1 contains 20 kg of mineral A and 40 kg of mineral B, and one bag of F_2 contains 30 kg each of mineral A and B, then how many bags of F_1 and F_2 should the farmers use to fulfill the requirement of both the types of minerals at an optimum low cost? Let us formulate this problem in terms of mathematical equations or inequalities. As the farmer has to decide on the number of bags of fertilizers F_1 and F_2 , the variables may be defined as: q_1 = number of bags of F_1 q_2 = number of bags of F_2 The objective function is minimization, that is, cost reduction. Here the total cost is $60q_1 + 80q_2$. The restriction is that at least 900 kg of mineral A and 1200 kg of mineral B is required. Hence we get the following constraints: $20q_1 + 30q_2$ < 900 — requirement for mineral A
Unit 14: Index Numbers 41 40

q 1 + 30q 2 < 1200 — requirement for mineral B As we cannot have negative quantities, q 1 < 0 and q 2 < 0, the problem may be represented as Minimize Z : 60q 1 + 80q 2 .....(1) Subject to constraints: 20q 1 + 30q 2 < 900 40q 1 + 30q 2 < 1200 .....(2) q 1 < 0, q 2 < 0 .....(3) We have to find the values of q 1 and q 2 which will satisfy constraints (2) and (3) and at the same time, minimize function (1). After formulating the problem, each inequality is converted to an equality. Then any arbitrary value (say, 0) is assigned to one variable in the equation and the corresponding value of the other variable is found. Consider the constraints which are written as equalities: 20q 1 + 30q 2 = 900. If q 1 = 0, we get q 2 = 30 and if q 2 = 0, we have q 1 = 45. These two points are now plotted on a graph with q 1 on X-axis and q 2 on Y-axis. Joining the two points (0, 30) and (45, 0), we get a straight line corresponding to the above equation. Consider the equation: 40q 1 + 30q 2 = 1200. If we take q 1 = 0, then q 2 = 40, and if q 2 = 0, then q 1 = 30. Joining the two points (0,40) and (30,0), we get another straight line corresponding to the above equation. The next step is to graph the feasible region which satisfies all the constraints. For this, we should take the co-ordinates of the point of origin (0,0) and substitute in each inequality. If the statement is found to be true, shade the region towards the origin or else shade the region away from the origin. Take the constraint 20q 1 + 30q 2 < 900. If we substitute (0,0), we get (20 x 0) + (30 x 0) < 900. Since the statement is not true, we shade the region away from the origin. Similarly, for constraint 40q 1 + 30q 2 < 1200, we shade the region away from the origin.
Figure 15.5
Block V: Advanced Statistics 42
The region which satisfies all the constraints is the feasible region. Here, the region above ABC (that is, the intersection of all shaded regions) is the feasible region. Now we should compute the co-ordinates of the corner points B, A and C of the feasible region. We know that the co-ordinates of B are (0,40) and that of C are (45,0). For point A, which is an intersection of the two straight lines of equations 20q 1 + 30q 2 = 900 and 40q 1 + 30q 2 = 1200, we find the co-ordinates by solving the simultaneous equations 20q 1 + 30q 2 = 900 (1) 40q 1 + 30q 2 = 1200 (2) Subtracting equation (1) from (2) we get 20q 1 = 300. Therefore, q 1 = 15 and q 2 = 20. Hence the co-ordinates of A are (15,20). The next step is to substitute the co-ordinates of the corner points of the feasible region in the objective function and choose the optimal solution (that is, the values that give the lowest cost). We thus get the following volumes: At
A (15, 20), Z = 15 x 60 + 20 x 80 = Rs.2,500, B (0, 40), Z = 0 x 60 + 40 x 80 = Rs.3,200 and C (45, 0), Z = 45 x 60 + 0 x 80 =
Rs.2,700 From the above calculations, we find that Z assumes a minimum value at A (15,20). Therefore, the optimal value of q 1 = 15 and q 2 = 20. Hence the farmer should buy 15 bags of fertilizer F 1 and 20 of fertilizer F 2 in order to meet the optimal requirements. Check Your Progress - 1 1. The relationship y = f(x) is defined as linear if, for all possible values of x and y, a given change in the value of x brings about a _______ change in the value of y. 2. In the general expression of a linear function y = a + bx, which is the independent variable? a. a b. b c. x d. y 3. In a
linear programming
problem, Maximize 20
x 1 +45x 2 +78x 3 (1) 12x 1 +23x 3 ≤90 13x 2 +32x 3 ≤80 11x 1 +12x 2 ≤70 .... (2) x 1 , x 2 , x 3 ≥ 0 (3)
Unit 14: Index Numbers 43 What are set of equations in2 referred as: a. Objective function b. Linear constraints c. Non negativity restrictions d. Linear equations 4. What will happen in a graphical method, if the objective function is parallel to an edge of the feasible region, a. No optimal solution b. Multiple optimal solutions c. One of the optimal solution may be fractional d. Not possible 5. How do you convert the objective function
Maximize 10

42%	MATCHING BLOCK 11/64	W

x 1 + 21 x 2 + 15 x 3 to a minimization function a. Minimize -(10x 1 + 21 x 2 + 15 x 3) b. Minimize - 10x 1 + 21 x 2 + 15 x 3 c. Minimize 10x 1 - 21 x 2 + 15 x 3 d. Minimize 10x 1 + 21 x 2 -15 x 3 15.6

37%	MATCHING BLOCK 12/64	W

x 1 + 21 x 2 + 15 x 3 to a minimization function a. Minimize -(10x 1 + 21 x 2 + 15 x 3) b. Minimize - 10x 1 + 21 x 2 + 15 x 3 c. Minimize 10x 1 - 21 x 2 + 15 x 3 d. Minimize 10x 1 + 21 x 2 -15 x 3 15.6 The

37%	MATCHING BLOCK 13/64	W

x 1 + 21 x 2 + 15 x 3 to a minimization function a. Minimize -(10x 1 + 21 x 2 + 15 x 3) b. Minimize - 10x 1 + 21 x 2 + 15 x 3 c. Minimize 10x 1 - 21 x 2 + 15 x 3 d. Minimize 10x 1 + 21 x 2 -15 x 3 15.6 The

Simplex Method In large sized linear programming problems, the solution cannot be obtained by the graphical method and hence a more systematic method has to be developed to find the optimal solution. The 'Simplex Method' developed by George B. Dantzig is an efficient algorithm to solve such problems. The simplex method is an iterative procedure for moving from an extreme point with a low profit value to another with a higher profit value until the maximum value of the objective function is achieved. An application of the simplex method is illustrated below with the problem solved by the graphical method. 15.6.1 Maximization Example 5 Maximize Z: 70q 1 + 40q 2 subject to: 2q 1 + q 2 > 120 0.8q 1 + 0q 2 > 40 3q 1 + 2q 2 > 200 4q 1 + 3q 2 > 360 q 1 < 0, q 2 < 0
Block V: Advanced Statistics 44 The first step in applying the simplex method is to convert all inequalities into equations. This conversion could be accomplished by utilizing the concept of slack or unused resources. If we define: S 1 = slack raw material-1 S 2 = slack raw material-2 S L = slack labor S C = slack foundry capacity Then the constraints will be: 2q 1 + q 2 + S 1 = 120 0.8q 1 + 0q 2 + S 2 = 40 3q 1 + 2q 2 + S L = 200 4q 1 + 3q 2 + S C = 360 q 1 < 0, q 2 < 0, S 1 < 0 S 2 < 0, S L < 0, S C ??0 The profit contribution of slacks is taken as 0 so that the objective function is 70q 1 + 40q 2 + 0S 1 + 0S 2 + 0S L + 0S C which is to be maximized. Let us call the original variables q 1 and q 2 as regular variables and the others as slack variables. We must first form an initial solution to the constraints. This is obtained by assigning the value '0' to the regular variables q 1 and q 2 , i.e., we shall start at the point 0 in the graph. The values of slack variables will now be: S 1 = 120 S 2 = 40 S L = 200 S C = 360 The non-flow
variables are called basic variables and the others are called non- basic variables (Basic:
S 1 , S 2 , S L , S C ; Non-basic: q 1 , q 2). We shall now construct the initial tableau and update it eventually. The explanation is given at the end of each table. Tableau 1 Profit C j 70 40 0 0 0 0 Variables q 1 q 2 S 1 S 2 S L S C Profit Variable Solution 1 2 3 4 5 6 7 8 9 0 S 1 120 2.0 1.0 1 0 0 0 0 S 2 40 0.8 0 0 1 0 0 0
S L 200 3.0 2.0 0 0 1 0 0 S C 360 4.0 3.0 0 0 0 1 Z j 0 0 0 0 0 0 (Z j – C j) –70 –40 0 0 0 0 * Profit = 0
Unit 14: Index Numbers 45
Explanation 1. Columns (4) to (9) represent all the variables that appear in the problem, that is, q 1 , q 2 , S 1 , S 2 , S L and S C . 2. First, fill in the second row with the variables. 3. Fill in the first row with the profit contribution of these variables. The profit contributions are the coefficients of the variables in the objective function. 4. Fill column (2) with the slack variables, S 1 , S 2 , S L , S C . 5. Fill column (1) with the profit contribution of variables that appear in column (2). 6. Fill column (3) with the solution values of the variables in column (2). 7. Column (4) is filled with coefficients of q 1 in the constraints. Similarly, columns (5) to (9) are filled with the coefficients of q 2 , S 1 , S 2 ,
S L and S C
respectively, in the constraints. 8. The value in the profit cell {column (3), last row} is obtained by multiplying the elements of column (1) with the elements of column (3), that is, 0 x 120 + 0 x 40 + 0 x 200 + 0 x 360 = 0 This implies that, for the initial solution q 1 = 0, q 2 = 0 (that is, no production), the profit realized is 0. 9. The cells in the last row under columns (4) to (9) are called (Z j – C j) cells. These values indicate the increase in the objective function per unit increase in the value of the variable currently at 0. In the initial tableau, this row is obtained by subtracting C j from Z j where Z j is the summation of products of each value in columns (4) to (9) and the value in column (1). For instance, the Z j value for column (4) is 2 x 0 + 0.8 x 0 + 3 x 0 + 4 x 0 = 0. For column (5), Z j is 1 x 0 + 0 x 0 + 2 x 0 + 3 x 0 = 0. Tableau 1 is now complete. At the end of each tableau, we should decide whether to update it to obtain a better solution, or to stop. This is done by the following steps: Step 1: Is the solution indicated in the tableau optimal? The answer to this question is obtained by looking at the (Z j – C j) values in the last row. As mentioned above, these values indicate the profit that could be gained by increasing the production levels. For example, the Z j – C j value corresponding to variable q 1 is –70. This indicates that, by not producing type A castings, the foundry has lost Rs.70 per unit and hence by increasing the production level of type A castings, the foundry can increase its profit at the rate of Rs.70 per unit increase in production. Similarly, the foundry can increase its profit by Rs.40 by increasing
Block V: Advanced Statistics 46

the production level of type B castings. Thus, as there is scope for improving the profit, we have not reached the optimal solution. Thus, we can answer the initial question by looking at all the (Z j − C j) values. If all these values are greater than or equal to zero, it implies that we have reached the optimal solution and hence we should stop. If one of the Z j − C j values is negative, then we should go to the next step. Step 2: Find the variable to 'enter solution' This means identifying the product for which the production level has to be increased. We have seen that it is optimal to increase the production level of q 1 . The rule is: Identify the least negative Z j − C j value. Thus the corresponding (non-basic) variables will increase in value. In this case, q 1 has been selected to be a basic variable.
Tableau 2 Profit C j 70 40 0 0 0 0 Variables q 1 q 2 S 1
S 2 S L S C Profit Variables Solution 1 2 3 4 5 6 7 8 9 0

61%	MATCHING BLOCK 18/64	W

S 1 120 − 0.8 40) x(2 = 20 0 0.8 0)x(2 1.0 ? = 1.0 0.8 0)x(2 1.0 ? = 1.0 0.8 1.0) x(2 0 ? = −2.5 0.8 0.0) x(2 0 ? = 0 ? (2x0) 0 0.8 = 0 70 q 1 50 0.8 40 ? 1 0.8 0.8 ? ? 0 0 0.8 ? 0 0 0.8 ? 1 1.25 0.8 ? 0 0 0.8 ? 0 0 0.8 0 S L 200 − 0.8 40)x (3.0 = 50 0 2.0 − 0.8 0)x(3 = 2.0 0 − 0.8 0)x(3 = 0 0 − 0.8 1)x(3 = −3.75 1.0 − 0.8 0)x(3 = 1.0 0 − 0.8 0)x(3 = 0 0 S C 360 − 0.8 40) x(4 = 160 0 3.0 − 0.8 0)x(4 = 3.0 0 − 0.8 0)x(4 = 0 0 − 0.8 1)x(4 = −5 0 − 0.8 0)x(4 = 0 1 − 0.8 0)x(4 = 1.0 Z j 70 0 0 87.5 0 0

68%	MATCHING BLOCK 15/64	W

x(2 = 20 0 0.8 0)x(2 1.0 ? = 1.0 0.8 0)x(2 1.0 ? = 1.0 0.8 1.0) x(2 0 ? = −2.5 0.8 0.0) x(2 0 ? = 0 ? (2x0) 0 0.8 = 0 70 q 1 50 0.8 40 ? 1 0.8 0.8 ? ? 0 0 0.8 ? 0 0 0.8 ? 1 1.25 0.8 ? 0 0 0.8 ? 0 0 0.8 0 S L 200 − 0.8 40)x (3.0 = 50 0 2.0 − 0.8 0)x(3 = 2.0 0 − 0.8 0)x(3 = 0 0 − 0.8 1)x(3 = −3.75 1.0 − 0.8 0)x(3 = 1.0 0 − 0.8 0)x(3 = 0 0 S C 360 − 0.8 40) x(4 = 160 0 3.0 − 0.8 0)x(4 = 3.0 0 − 0.8 0)x(4 = 0 0 − 0.8 1)x(4 = −5 0 − 0.8 0)x(4 = 0 1 − 0.8 0)x(4 = 1.0 Z

66%	MATCHING BLOCK 16/64	W

x(2 = 20 0 0.8 0)x(2 1.0 ? = 1.0 0.8 0)x(2 1.0 ? = 1.0 0.8 1.0) x(2 0 ? = −2.5 0.8 0.0) x(2 0 ? = 0 ? (2x0) 0 0.8 = 0 70 q 1 50 0.8 40 ? 1 0.8 0.8 ? ? 0 0 0.8 ? 0 0 0.8 ? 1 1.25 0.8 ? 0 0 0.8 ? 0 0 0.8 0 S L 200 − 0.8 40)x (3.0 = 50 0 2.0 − 0.8 0)x(3 = 2.0 0 − 0.8 0)x(3 = 0 0 − 0.8 1)x(3 = −3.75 1.0 − 0.8 0)x(3 = 1.0 0 − 0.8 0)x(3 = 0 0 S C 360 − 0.8 40) x(4 = 160 0 3.0 − 0.8 0)x(4 = 3.0 0 − 0.8 0)x(4 = 0 0 − 0.8 1)x(4 = −5 0 − 0.8 0)x(4 = 0 1 − 0.8 0)x(4 = 1.0 Z

66%	MATCHING BLOCK 17/64	W

x(2 = 20 0 0.8 0)x(2 1.0 ? = 1.0 0.8 0)x(2 1.0 ? = 1.0 0.8 1.0) x(2 0 ? = −2.5 0.8 0.0) x(2 0 ? = 0 ? (2x0) 0 0.8 = 0 70 q 1 50 0.8 40 ? 1 0.8 0.8 ? ? 0 0 0.8 ? 0 0 0.8 ? 1 1.25 0.8 ? 0 0 0.8 ? 0 0 0.8 0 S L 200 − 0.8 40)x (3.0 = 50 0 2.0 − 0.8 0)x(3 = 2.0 0 − 0.8 0)x(3 = 0 0 − 0.8 1)x(3 = −3.75 1.0 − 0.8 0)x(3 = 1.0 0 − 0.8 0)x(3 = 0 0 S C 360 − 0.8 40) x(4 = 160 0 3.0 − 0.8 0)x(4 = 3.0 0 − 0.8 0)x(4 = 0 0 − 0.8 1)x(4 = −5 0 − 0.8 0)x(4 = 0 1 − 0.8 0)x(4 = 1.0 Z

73%	MATCHING BLOCK 19/64	SA	Assignment+2+Josep+Grau+Miro%CC%81.docx (D42192805)

x(2 = 20 0 0.8 0)x(2 1.0 ? = 1.0 0.8 0)x(2 1.0 ? = 1.0 0.8 1.0) x(2 0 ? = −2.5 0.8 0.0) x(2 0 ? = 0 ? (2x0) 0 0.8 = 0 70 q 1 50 0.8 40 ? 1 0.8 0.8 ? ? 0 0 0.8 ? 0 0 0.8 ? 1 1.25 0.8 ? 0 0 0.8 ? 0 0 0.8 0 S L 200 − 0.8 40)x (3.0 = 50 0 2.0 − 0.8 0)x(3 = 2.0 0 − 0.8 0)x(3 = 0 0 − 0.8 1)x(3 = −3.75 1.0 − 0.8 0)x(3 = 1.0 0 − 0.8 0)x(3 = 0 0 S C 360 − 0.8 40) x(4 = 160 0 3.0 − 0.8 0)x(4 = 3.0 0 − 0.8 0)x(4 = 0 0 − 0.8 1)x(4 = −5 0 − 0.8 0)x(4 = 0 1 − 0.8 0)x(4 = 1.0

Z j − C j 0 −40 0 87.5 0 0 *
Profit = 70 x 50 = 3500

Step 3: Find the variable to 'leave solution' This is obtained by answering the following question: In Step 2, let us decide to increase the value of q 1 , that is, increase the production level of type A castings. By how much can the production of type A castings be increased?
Unit 14: Index Numbers 47 The answer is this: Increase the production until one of the resources gets exhausted. The requirement per unit of q 1 is given in Column (4) of the tableau. If the value is positive, it means that there is a positive requirement. If the value is 0 or negative, it means that particular resources are not required for increasing the value of q 1 . By applying this simple logic, it is observed that the minimum positive ratio of the elements of Column (3) with the elements of the column selected in Step 2, that is, column (4) will indicate the resource which will be the first to be exhausted. ? ? ? ? ? ? 4.0 360 , 3.0 200 , 0.8 40 , 2.0 120
Min
The minimum value is 50 corresponding to the ratio 40/0.8 and the corresponding variable is S 2 , that is, the resource raw material 2, gets exhausted by producing 50 units of type A castings. We cannot increase the value of q 1 beyond 50 at this stage. As the value of S 2 reduces to 0, we replace S 2 by q 1 in Column (2). We then identify the row corresponding to S 2 . Step 4: Identify the pivot element This is the one element common to
the column identified in Step 2 and the row identified in Step 3, that is, 0.8.
This element is circled in the table. The column and row where the pivot element lies are called pivot column and pivot row respectively. We are now ready to update Tableau 1 and construct Tableau 2. The format of Tableau 2 is the same as Tableau 1. The rules for updating are explained below in Tableau 2. Explanation 1. Fill in Row 1 and Row 2 of Tableau 2 in the same way as Tableau 1. Fill in Column (2) by replacing the variable selected in Step 3 with the variable selected in Step 2. 2. Fill in Column (1) with the profit contribution of the variables in Column 2. 3. Update the pivot row by dividing each element by the pivot element. 4. Update the pivot column by filling it with zeros. Note that the value of the key element in an updated table will be 1 and the values of the key or pivot column will be zero. Also, if a certain value in the pivot column is 0, then all the values in the corresponding row of the updated table remains the same as in the previous table. If the key row contains a 0, then the values of the corresponding columns in the updated table will remain the same as in the previous table. For example, we have 0 in the key row of Tableau 1 which corresponds to columns 5, 6, 8 and 9. One can notice that in Tableau 2, these columns retain the values of Tableau 1. 5. Update all other elements as follows:
Block V: Advanced Statistics 48 The first element in column 3 is 120 and it is to be updated. From this element we move to the pivot column, then to the pivot and then to the column we started with and trace the elements. These are: Value to be updated 120 2.0 40 0.8 Pivot The Updated value = Old value –
element Pivot element) diagonal the of (Product or Updated value = Old value – ? ? ? ? ? ? ? ? ? ? ? ? ? ? element Pivot column key previous in no. row old ing Correspond x row key previous the in value ing Correspond = 120 – = 20
Similarly,
the third, fourth and the profit values in column 3 are updated as follows: Value to be updated 40 200 0.8 3.0 Pivot new value = 200 – (3.0x40) 0.8 = 50
Value
to be updated 40 360 0.8 4.0 Pivot new value = 360 – (4.0x40) 0.8 = 160 Value to be updated 40 0 0.8 –70 Pivot new value = 0 – (- 70)x 40 0.8 = 3,500 The other elements are also updated in a similar fashion. The calculations are shown in the tableau. The solution indicated in Tableau 2 is (from columns 2 and 3): S 1 = 20 q 1 = 50 S L = 50 S C = 160 The variables q 2 and S 2 which do not appear in Column 2 take a value 0. Notice that the solution q 1 = 50, q 2 = 0 corresponds to the point D in the graph. Thus, we moved from point 0 to point D in the graph by updating Tableau 1. 0.8 40) x (2.0
Unit 14: Index Numbers 49 At the end of Tableau 2, the four steps mentioned at the end of Tableau 1 are repeated. These are: Step 1: Is the solution indicated in Tableau 2 optimal? The (Z j – C j) values are now (0, –40, 0, 87.5, 0, 0) and the negative values present it imply that we have not yet reached the optimal solution. Step 2: Find the variable to enter the solution. The negative (Z j – C j) value is –40 and this corresponds to the variable q 2 . Hence, q 2 is the variable which enters the solution in the next iteration. Step 3: Find the variable to leave the solution. The ratios to be considered are: ? ? ? ? ? ? 3.0 160 , 2.0 50 , 1.0 20 The ratio corresponding to q 1 is ignored as the corresponding value in column 5 is 0. The minimum ratio is 20 corresponding to 20/1.0. Hence, the variable S 1 leaves the solution, and becomes non-basic. Step 4: Identify the pivot element. From steps 3 and 4, we find that the pivot element is 1.0, the first element in column 5, and this is circled. We are now ready to construct Tableau 3. Tableau 2 is updated using the updating rules. Subsequent tableaus are also constructed and shown below:

Tableau 3 Profit C j 70 40 0 0 0 0 Variables q 1 q 2 S 1 S 2 S L S C Profit Variables Solution 1 2 3 4 5 6 7 8 9 40 q 2 20 0 1 1.0 −2.5 0 0 70 q 1 50 1 0 0 1.25 0 0 0 S L 10 0 0 −2.0 1.25 1 0 0 S C 100 0 0 −3.0 2.5 0 1 Z j 70 40 40 −12.5 0 0 Z j − C j 0 0 40 −12.5 0 0 * Profit = 40 x 20 + 70 x 50 = 4300 Pivot is 1.25.

The solution is q 1 = 50, q 2 = 20 corresponds to point C in the graph, which is not optimal.

Block V: Advanced Statistics 50 Tableau 4 Profit C j 70 40 0 0 0 0 Variables q 1 q 2 S 1 S 2 S L S C Profit Variables Solution 1 2 3 4 5 6 7 8 9 40 q 2 40 0 1 −3 0 2 0 70 q 1 40 1 0 2 0 −1 0 0 S 2 8 0 0 −1.6 1 0.8 0 0 S C 80 0 0 1 0 −2 1 Z j 70 40 20 0 10 0 Z j − C j 0 0 20 0 10 0 * Profit = 70 x 40 + 40 x 40 = 4400

The optimal solution is reached as all (Z j − C j) values are non-negative. The optimal solution is: q 1 = 40 q 2 = 40 S 2 = 8 S C = 80 S 1 = 0 S L = 0 The maximum profit is Rs.4,400, that is, produce 40 of type A castings and 40 of type B castings and we are left with 8 kg. of raw material-2 and 80 units of unused foundry capacity. Raw material-1 and labor will be completely utilized. Example 6 Consider a firm producing batteries for cars and trucks. Each car battery costs Rs.400 in materials and machine time plus Rs.200 in wages and each truck battery costs Rs.650 in materials and machines plus Rs.150 in wages. The selling prices of a car and a truck battery are Rs.700 and Rs.1,000 respectively. The firm is able to sell as many units as it can produce. The firm wants to plan its next months' production. The firm has 2,100 hours of machine time and 1,000 hours of assembly time available in the next month. The production of each car battery requires 10 hours of machine time and 10 hours of assembly time. The production of each truck battery requires 30 hours of machine time and 10 hours of assembly time. The firm arrived at a forecast of the cash balance of Rs.72,000, which can be used to meet the expenses of materials and wages. Assume that the payment for materials and wages has to be made in the month and the firm cannot get any more cash. With these production and financial constraints, the firm has to determine the product mix which will give maximum profit next month. Let x and y be the number of car and truck batteries respectively, to be produced next month. The profit on a car battery is sales price minus the cost of materials

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and wages, that is, 700 − 400 − 200 = 100. Similarly, the profit on a truck battery is 1,000 − 650 − 150 = 200. The problem is to find the values of x and y that will maximize the profit. The formulation is: Maximize 100x + 200y Subject to: Machine capacity : 10x + 30y ? 2,100 Assembly capacity : 10x + 10y ?? 1,000 Cash availability : 600x + 800y ??72,000 x ? 0, y ? 0 For applying simplex method, we convert the inequalities into equalities by adding

the

slack variables S 1 , S 2 , and S 3 for the constraints. The problem now is:

97% MATCHING BLOCK 20/64 W

Maximize 100x + 200y + 0S 1 + 0S 2 + 0S 3 Subject to: 10x + 30y + S 1 = 2,100 10x + 10y + S 2 = 1,000 600x + 800y + S 3 = 72,000 x ? 0, y ? 0, S 1 ? 0, S 2 ? 0, S 3 < 0

The

tableaus are constructed below to obtain the optimal solution. The pivots identified in each tableau are circled. Tableau 1 Profit 100 200 0 0 0 Variables x y S 1 S 2 S 3 Profit Variables Solution 0 S 1 2,100 10 30 1 0 0 0 S 2 1,000 10 10 0 1 0 0 S 3 72,000 600 800 0 0 1 Z j − C j 0 −100 −200 0 0 0 Tableau 2 Profit 100 200 0 0 0 Variables x y S 1 S 2 S 3 Profit Variables Solution 200 y 70 1 3 1 1 30 0 0 0 S 2 300 20 3 0 − 1 3 1 0 0 S 3 16,000 1,000 3 0 − 80 3 0 1 Z j − C j 14,000 -100 3 0 20 3 0 0

Block V: Advanced Statistics 52 Tableau 3 Profit 100 200 0 0 0 Variables x y S1 S2 S3 Profit Variables Solution 200 Y 55 0 1 1 20 − 1 20 0 100 X 45 1 0 1 -20 3 20 0 0 S 3 1,000 0 0 −10 −50 1 Z j − C j 15,500 0 0 5 5 0 The optimal solution is: x = 45 and y = 55 and the maximum profit = 15,500, that is, the firm needs to produce 45 car batteries and 55 truck batteries. 15.6.2

Minimization Example 7 Let us consider the minimization problem solved graphically earlier. Minimize Z = 60q 1 + 80q 2 subject to constraints 20q 1 + 30q 2 < 900 40q 1 + 30q 2 < 1200 q 1 < 0, q 2 < 0 The solution for minimization is similar to that of maximization except that, we introduce some new variables to convert inequalities into equalities. The variable we use to convert the 'greater than' type of inequality into an equation is called 'surplus variable' and it represents the excess of what is generated over the requirement. To convert the inequality to an equation, we should subtract the surplus variable from the LHS. Now we have, 20q 1 + 30q 2 – S 1 = 900 40q 1 + 30q 2 – S 2 = 1200 If the values of q 1 and q 2 are equal to zero, we get S 1 = – 900 and S 2 = –1200. This, however, is not feasible as it violates the non-negativity restriction. To avoid this we add artificial variables to the LHS. These artificial variables do not represent any quantity relating to the decision problem hence, they must be removed in the last solution. If they appear in the final solution, it represents a situation of infeasibility. To ensure that this does
Unit 14: Index Numbers 53 not take place, we assign a value M, which is very large, to each artificial variable. M represents a number higher than any finite number. Hence this method is also called the Big M method. Now the objective function and subject to constraints are written as

38% MATCHING BLOCK 21/64 W

Minimize Z = 60q 1 + 80q 2 + 0S 1 + 0S 2 + MA 1 + MA 2 Subject to constraints 20q 1 + 30q 2 – 1S 1 + 0S 2 + 1A 1 + 0A 2 = 900 40q 1 + 30q 2 + 0S 1 – 1S 2 + 0A 1 + 1A 2 = 1200 The initial simplex

38% MATCHING BLOCK 22/64 W

Minimize Z = 60q 1 + 80q 2 + 0S 1 + 0S 2 + MA 1 + MA 2 Subject to constraints 20q 1 + 30q 2 – 1S 1 + 0S 2 + 1A 1 + 0A 2 = 900 40q 1 + 30q 2 + 0S 1 – 1S 2 + 0A 1 + 1A 2 = 1200 The initial simplex

38% MATCHING BLOCK 23/64 W

Minimize Z = 60q 1 + 80q 2 + 0S 1 + 0S 2 + MA 1 + MA 2 Subject to constraints 20q 1 + 30q 2 – 1S 1 + 0S 2 + 1A 1 + 0A 2 = 900 40q 1 + 30q 2 + 0S 1 – 1S 2 + 0A 1 + 1A 2 = 1200 The initial simplex

table is given below. Note how we introduce artificial variables first in the initial solution. Table 1 Cost C j 60 80 0 0 M M Minimum Ratio Key Row Variables q 1 q 2 S 1 S 2 A 1 A 2 Basic Variable Co- efficient of basic variable Solution values

47% MATCHING BLOCK 26/64 W

A 1 M 900 20 30 –1 0 1 0 900/20 A 2 M 1200 40 30 0 –1 0 1 1200/40 Z j 60M 60M –M –M M M C j – Z j 60– 60M 80– 60M M M 0 0 Key Column

73% MATCHING BLOCK 24/64 W

M 900 20 30 –1 0 1 0 900/20 A 2 M 1200 40 30 0 –1 0 1 1200/40 Z j 60M 60M –M –M M M C j – Z j 60– 60M 80– 60M M M 0 0

73% MATCHING BLOCK 25/64 W

M 900 20 30 –1 0 1 0 900/20 A 2 M 1200 40 30 0 –1 0 1 1200/40 Z j 60M 60M –M –M M M C j – Z j 60– 60M 80– 60M M M 0 0

Explanation (Table 1) 1. In a minimization problem, we compute C j – Z j , instead of Z j – C j , as in the case of a maximization problem. The rest of the procedure remains the same. 2. Finding the variable to 'enter solution': This is done by identifying the key column (K.C). The column corresponding to the most negative value in the (C j – Z j) row is called the key column in table 1. 3. Find the variable to 'leave solution': This is done by identifying the key row. As in a maximization problem, the row corresponding to the minimum positive ratio is the key row where the minimum ratio is calculated by dividing the solution values with corresponding values of the key column. 4. Optimal level is reached when the M values, if any, in the (
C j – Z j) row are positive.
Block V: Advanced Statistics 54
Table 2 Cost C j 60 80 0 0 M M Minimum Ratio Key Row Variables q 1 q 2 S 1 S 2 A 1 A 2 Basic Variable Co- efficient of basic variable Solution values

63%	MATCHING BLOCK 27/64	W

A 1 M 300 0 15 –1 1/2 1 –1/2 20 q 1 60 30 1 3/4 0 –1/40 0 1/40 40 Z j 60 15M+ 45 –M M/2 – 3/2 M –M/2 + 3/2 C j – Z j 0 35– 15M M 3/2– M/2 0 – 3 2 + 3M 2

63%	MATCHING BLOCK 28/64	W

A 1 M 300 0 15 –1 1/2 1 –1/2 20 q 1 60 30 1 3/4 0 –1/40 0 1/40 40 Z j 60 15M+ 45 –M M/2 – 3/2 M –M/2 + 3/2 C j – Z j 0 35– 15M M 3/2– M/2 0 – 3 2 + 3M 2

Key Column Explanation (Table 2) 1. The values in table 2 are updated in the same manner as in a maximization problem. 2. For example, Z j value of q 2 column is computed as 15 x M + 60 x 3/4 = 15M + 45 3. The corresponding C j – Z j value is 80 – (15M + 45) = –15M + 35 4. The variable which corresponds to the most negative (C j – Z j) value is the one that enters the next solution. Here the most negative C j – Z j value is 35 – 15M and the corresponding variable that enters the next solution is q 2 . 5. The variable that leaves the solution is the one corresponding to the least positive ratio. Here the minimum positive ratio is 20 and the corresponding variable that leaves the solution is A 1 . Table 3 Cost C j 60 80 0 0 M M Minimum Ratio Variables q 1 q 2 S 1 S 2 A 1 A 2 Basic Variable Coefficient of basic variable Solution values q 2 80 20 0 1 –1/15 1/30 1/15 –1/30 q 1 60 15 1 0 1/20 –1/20 –1/20 1/20 Z j 60 80 –7/3 –1/3 7/3 1/3 C j – Z j 0 0 7/3 1/3 M – 7/3 M – 1/3 As all C j – Z j values are non-negative, we have reached the optimal solution. The optimal solution is q 1 = 15 and q 2 = 20. The lowest cost is calculated by substituting the solution values in the objective function. Minimize Z = 60 x 15 + 20 x 80 = 2500
Unit 14: Index Numbers 55 The least cost is Rs.2500. Some Scenarios in the Simplex Method In this section, we are dealing with some different scenarios that may occur in the simplex method. A. Degeneracy. B. Alternative Optima C. Unbounded Solutions D. Infeasible Solutions The above scenarios occur in different stages (Initialization, Iteration, and Termination) of the Simplex method. Degeneracy occurs in the iteration phase of the simplex method, whereas the remaining three scenarios occur at the termination phase. A. Degeneracy: Degeneracy occurs when there is a tie for the leaving variable or, for the minimum ratio. This tie can be broken arbitrarily. This tie in the decision of leaving variable results for atleast one basic variable to have a zero value in the next iteration. The solution under this circumstance is known as Degenerate Solution. The phenomenon is known as Degeneracy. Degeneracy results extra iteration in the simplex method but it doesn't improve the objective function value. B. Alternate Optima: The alternate optimum occurs when the objective function is parallel to the no redundant binding constraint. No redundant binding constraint means at the optimal point the constraint is satisfied as an equation. The objective function has same objective function value (optimal) at more than one point.
There are

infinite numbers of alternate optimum can occur in a problem but in case of simplex method only two alternate solutions are highlighted corresponding to the corner points. In Simplex method, alternate optimum occurs when the Cj-Z j value of a non-basic variable become zero and the variable enter into the simplex iteration. We get different points where the optimal value of the objective function is same. We consider an example and try to illustrate the alternate optimum by using the graphical approach. Suppose, the objective function Z is $3X_1 + 6X_2$ and the function type is maximization. Also, we have two constraints such as $X_1 + X_2 \leq 3$ and $2X_1 + 4X_2 \leq 6$. Solving the problem by using the graphical approach, we get two corner points A (0,1.5) and C (3,0). The objective function value at these two corner points is 9. The objective function Z is parallel to the non- redundant second constraint i.e. $2X_1 + 4X_2 \leq 6$. Any point on the line segment AC represents an alternate optimum with an objective function value of 9.
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C. Unbounded Solution: The unbounded solution arises when the solution space is unbounded in atleast one variable (without violating any constraints). Under this circumstance, both the solution space (and feasible space) and the objective function value are unbounded in nature. Unboundness situation may arise due to the poor estimation of the parameters in the model. In Simple method, Unboundness happens when we are unable to find any leaving variable and the simplex algorithm terminates. Sometimes due to the nature of the objective function, we still get an optimal solution even if the solution space is unbounded. For example, the objective function Z is $3X_1 + 6X_2$ and the function type is maximization. There are two constraints such as $X_1 - 2X_2 \leq 4$ and $X_1 \leq 20$. From the graphical method, we can observe that the solution space formed by these two constraints are not bounded. As the objective function type is maximization, we are not able to find the optimal solution. This phenomenon is known Unboundness. D. Infeasible Solution: Infeasibility occurs when there is no feasible solution space. Infeasibility may not occur, if all the constraints are less than equal type and non-negative right- hand side value. Sensitivity Analysis In this section, we discuss the variation in the model parameters without effecting the optimal value of the model. This is known as Sensitivity Analysis. In the next section we discuss the post optimal analysis where we will determine new optimal solution by changing the model parameters. To discuss sensitivity analysis in this section, we mainly focus on graphical approach. Sensitivity analysis can be performed in two ways: a. Sensitivity analysis with respect to the changes in resource availability (Right hand side Value) and b. Sensitivity analysis with respect profit and cost. Let us consider two examples, to illustrate the above cases. Sensitivity analysis with respect to the changes in the resource availability Let consider,

68% **MATCHING BLOCK 29/64** **W**

a company produces two products (A and B) on two different machines. One unit of product A takes 3 hours

in the first machine and 4 hours in the second machine, whereas the one unit of product B takes 2 hours in the first machine and 3 hours in the second machine. The total availability of the both machines is 6 hours daily. The unit profit for both products are Rs. 40 and Rs. 30 respectively. Based on the above scenario, we formulate the LP model where the objective function (maximization) Z is $40X_1 + 30X_2$, followed by the constraints $3X_1 + 2X_2 \leq 6$ and $2X_1 + 3X_2 \leq 6$. X_1 and X_2 represent the quantity of product A and B respectively and nonnegative. Solving the problem by using
Unit 14: Index Numbers 57
the graphical approach, we get the optimal value at point D (1.2, 1.2) and the optimum value of Z is 84. Now, if we increase the daily capacity of the first machine by one unit (the capacity of the second machine remains constant), then new optimal solution occurs at (1.8, 0.8) and the optimal value of Z is 96. Thus, rate of
the
change in objective function value with respect to change in capacity of first machine is (96-84) / (7-6) = 12. Similarly, if we decrease the capacity of the first machine by one unit, the rate of change of the objective function will remain same. Thus, we can say that unit increase (or decrease) of the first machine capacity will increase (or decrease) the profit by Rs. 14. This is known as dual price or Shadow price of the first machine. Similarly, if we perform the same process for the second machine, we will observe that
the rate of change of the objective function value with respect to

the change in capacity of the second machine is 2. Thus, the dual price of the second machine is 2. Sensitivity Analysis with respect to unit price or profit Consider the same example, the optimal value of the objective function Z remains same until the objective function lies between the two constraints. Suppose, the objective function is Z= C 1 X 1 + C 2 X 2 and the constraints are same as above. The optimal value of Z remains same if 2/3 ≤ (C 1 /C 2) ≤3/2. 15.7 Post-Optimal Analysis It can be seen from the optimal solution for the foundry problem that two resources, raw material-1 and labor, are exhausted whereas the other two resources, raw material-2 and foundry capacity, remain available. This implies that the availability of raw material-1 and labor are both exerting a restrictive effect on foundry operation and its profitability. Let us suppose, the foundry can buy the raw material-1 in the open market at a cost of Rs.15 per kg, then is it worth buying and increasing its production to make more profit? Similarly, if the foundry can hire extra labor for Rs.15 per day, then is it worth hiring the extra labor? The above questions can be answered from the (
Z j − C j) values in the final tableau. The (Z j − C j)
value corresponding to variables S 1 , that is, column 6 is 20. This indicates that the profit can be increased by Rs.20 for a unit increase in the availability of raw material-1. Thus, if one kilogram of raw material-1 costs Rs.15, then by purchasing it and changing the product mix, the foundry can increase its profit by Rs.20 − Rs.15 = Rs.5. Similarly, the (Z j − C j) value corresponding to the variable S L is 10 and this indicates that for a unit increase in the availability of labor, profit can be increased by Rs.10. Hence, it is worth hiring labor if its cost is less than Rs.10. Similarly, it is worth buying raw material-1 from open market, if its cost is less than Rs.20 per kg. The information in the final tableau is also useful in studying the effects of the variations in the profit contributions on the product mix.
Block V: Advanced Statistics 58 15.8
Duality For every LP formulation there exists another unique linear programming formulation called the 'Dual' (the original formulation is called the 'Primal'). Same data can be used for both 'Dual' and 'Primal' formulation. Both can be solved in a similar manner as the Dual is also an LP formulation. The Dual can be considered as the 'inverse' of the Primal in every respect. The column coefficients in the Primal constraints become the row co-efficients in the Dual

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constraints. The coefficients in the Primal objective function become the right-hand-side constraints in the Dual constraints. The column of

constants on the right hand side of the Primal constraints becomes the row of coefficients of the dual objective function. The direction of the inequalities are reversed. If the primal objective function is a 'Maximization' function then the dual objective function is a 'Minimization' function and vice versa.
Example 8 Consider
the following 'Primal' LP formulation.
Maximize 12

56% MATCHING BLOCK 32/64 W

x 1 + 10x 2 subject to 2x 1 + 3x 2 >?18 2x 1 + x 2 ??14 x 1 , x 2 < 0 The 'Dual' formulation for this problem would be Minimize 18y 1 + 14y 2 subject to 2y 1 + 2y 2 ? 12 3y 1 + y 2 ??10 y 1 ?? 0, y 2 <??0

95% MATCHING BLOCK 33/64 W

Minimize 18y 1 + 14y 2 subject to 2y 1 + 2y 2 ? 12 3y 1 + y 2 ??10 y 1 ?? 0, y 2 <??0

100% MATCHING BLOCK 31/64 W

y 1 + 14y 2 subject to 2y 1 + 2y 2 ? 12 3y 1 + y 2 ??10 y 1 ?? 0, y 2 <??0

Note the following: 1. The column coefficient in the Primal constraint namely (2,2) and (3,1) have become the row coefficient in the Dual constraints. 2. The coefficient of the Primal objective function namely, 12 and 10 have become the constants in the right-hand-side of the Dual constraints. 3. The constants of the Primal constraints, namely 18 and 14, have become the coefficient in the Dual objective function. 4. The direction of the inequalities have been reversed. The Primal constraints have the inequalities of > while the Dual constraints have the inequalities of <. 5. While the Primal is a 'Maximization' problem the Dual is a 'Minimization' problem and vice versa.
Unit 14: Index Numbers 59 15.8.1 Why the Dual Formulation? Dual formulation is done for a number of reasons. The solution to a Dual problem provides all essential information about the solution to the Primal problem. A solution for the LP problem can be determined either by solving the original problem or the Dual problem. Sometimes it may be easier to solve the Dual problem rather than the Primal problem as when the primal involves few variables but many constraints. Remark In the above sections, we have learnt to apply the simplex method to problems where the objective function is to be maximized. What if the problem were to Minimize 50x – 70y? This is equivalent to – {Maximize – (50x – 70y)} = – {Maximize −50x + 70y} We apply the Simplex Method and the final solution is obtained by taking the negative of the optimum solution (of the Maximization problem). Comparing the Optimal Solutions of the Primal and Dual Let us consider the example discussed under minimization. If it is considered as Primal, then the Dual is a maximization problem.

38% **MATCHING BLOCK 34/64** **W**

Primal Dual Minimize Z Maximize Z = 60q 1 + 80q 2 = 900y 1 + 1200y 2 Subject to Subject to 20q 1 + 30q 2 < 900 20y 1 + 40y 2 > 60 40q 1 + 30q 2 < 1200 30y 1 + 30y 2 > 80 q 1 , q 2 < 0 y 1 , y 2 < 0

38% **MATCHING BLOCK 35/64** **W**

Primal Dual Minimize Z Maximize Z = 60q 1 + 80q 2 = 900y 1 + 1200y 2 Subject to Subject to 20q 1 + 30q 2 < 900 20y 1 + 40y 2 > 60 40q 1 + 30q 2 < 1200 30y 1 + 30y 2 > 80 q 1 , q 2 < 0 y 1 , y 2 < 0

The simplex table containing the optimal solution to the primal which was discussed earlier is as follows.
Simplex Table: Optimal Solution of the Primal Cost C j 60 80 0 0
M M Variables q 1 q 2 S 1 S 2 A 1 A 2 Basic Variable Coefficient of basic variable Solution values q 2 80 20 0 1 −1/15 1/30 1/15 −1/30 q 1 60 15 1 0 1/20 −1/20 −1/20 1/20
Z j 60 80 −7/3 −1/3 7/3 1/3 C j – Z j 0 0 7/3 1/3 M – 7/3 M – 1/3 Let us now consider the solution to the Dual problem.
Block V: Advanced Statistics 60 Introducing slack variables in the corresponding maximization problem,
we get

55% **MATCHING BLOCK 36/64** **W**

Maximize Z = 900y 1 + 1200y 2 + 0S 1 + 0S 2 Subject to 20y 1 + 40y 2 + 1S 1 + 0S 2 = 60 30y 1 + 30y 2 + 0S 1 + 1S 2 = 80 In the initial solution

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y 1 + 1200y 2 + 0S 1 + 0S 2 Subject to 20y 1 + 40y 2 + 1S 1 + 0S 2 = 60 30y 1 + 30y 2 + 0S 1 + 1

we introduced the basic variables S 1 and S 2 (as shown in the following Table 1). On comparing the optimal solutions of the Primal and Dual problems it may be noted that there is a correspondence between their variables. In effect the following observations are made: a. The optimal solution objective function value is the same for both Primal and Dual. q 1 = 15 and q 2 = 20, therefore minimize Z = 15 x 60 + 80 x 20 = 2500. Similarly y 1 = 7/3 and y 2 = 1/3, therefore maximize Z = 900 x 7/3 + 1200 x 1/3 = 2500. Table 1 Profit C j 900 1200 0 0 Key Row Variables y 1 y 2 S 1 S 2 Basic Variable Coefficient of basic variables Solution values Minimum Ratio S 1 0 60 20 40 1 0 3/2 = 1.5 S 2 0 80 30 30 0 1 8/3 = 2.6 Z j 0 0 0 0 Z j − C j −900 −1200 0 0 Key Column Table 2 Profit C j 900 1200 0 0 Key Row Variables y 1 y 2 S 1 S 2 Minimum Ratio Basic Variable Coefficient of basic variables Solution values y 2 1200 3/2 1/2 1 1/40 0 3 S 2 0 35 15 0 −3/4 1 7/3 Z j 600 1200 30 0 Z j − C j −300 0 30 0 Key Column Table 3: Optimal Solution of the Dual Profit C j 900 1200 0 0 Variables y 1 y 2 S 1 S 2 Basic Variable Coefficient of basic variable Solution values y 2 1200 1/3 0 1 1/20 −1/30 y 1 900 7/3 1 0 −1/20 1/15 Z j 900 1200 15 20 Z j − C j 0 0 15 20
Unit 14: Index Numbers 61 b. The optimal solution values

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of the dual variables (7/3, 1/3) are the coefficients of the slack variables in the ? j row of the

primal. Thus, in the Dual y 1 = 7/3 and y 2 = 1/3, whereas in the Primal S 1 = 7/3 and S 2 = 1/3 in the ? j row. As we have seen ? j = Z j − C j for any maximization problem, whereas ? j = C j − Z j for any minimization problem. c. Values in the ? j row under columns S 1 and S 2 of the optimum table of the Dual are the same as values corresponding to the solution variables q 1 and q 2 in the optimal table of Primal. Hence, we can conclude that optimum solution to a primal can always provide solution to its Dual and vice-versa. Hence both Primal and Dual need not be solved to obtain the solution. This is a computational advantage in some situations. 15.8.2
Integer Programming An integer programming problem is identical to a linear programming problem except that one or more decision variables are constrained to take integer values. Such problems cannot be solved by the simplex method. They are solved by specialized procedures which are computer intensive. 15.8.3 Goal Programming This provides a more realistic model. In a modern setting, profit maximization may not be the only objective of a business concern. Other objectives or goals could be sound ecological management, networking in the neighborhood and maximizing market share. These goals may dominate the earlier objective. The idea is that a decision maker may not always be searching for an optimal solution but a “satisfying” solution that attempts to satisfy the many concerns of the management. Prof. Herbert A. Simon felt that a manager may not be able to optimize but may have to “satisfy”. Simplex method requires one goal. The goals are weighted and a single objective function is constructed which is then optimized to solve a goal programming problem. 15.9
The Transportation Problem The transportation problem is a special case of linear programming. In the general form, it has a number of destinations. A certain quantity of commodity is produced at each origin and it is to be transported to destinations, each of which has certain requirements. The objective of the problem is to meet the requirements of the destination with supply from the sources and to ensure that the transportation costs are minimal. This method can be applied to situations which involve the

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physical movement of goods from plants to warehouses, warehouses to wholesalers, wholesalers to retailers, and from retailers to

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from plants to warehouses, warehouses to wholesalers, wholesalers to retailers, and

Block V: Advanced Statistics 62

customers. These models can also be applied to production scheduling and inventory control. Such models are preferred as they reduce the computational effort involved in the simplex method. A transportation problem can be either balanced or unbalanced. It is said to be balanced if the quantity of goods produced is equal to the total requirement of all the warehouses. Otherwise it is considered as unbalanced. In an unbalanced problem, a dummy warehouse is added if the production capacity is more than the requirement; if the production capacity is less than the requirement a dummy origin is added with the desired quantity to make it a balanced one. The transportation problem can be formulated as a linear programming problem as shown: X ij is the quantity transported from plant P i to a warehouse

W j . C ij

is the unit transportation cost from P i to W j . As

the objective of a transportation problem is to minimize the total transportation cost,

the objective function can

38%	MATCHING BLOCK 39/64	W

be given as Minimize ? ? ij X ij C Z Subject to the supply constraints, 2...m 1, i and S X i n 1j ij ? ? ? ? Demand constraints, 2...n 1, j and D X j m 1i ij ? ? ? ?

Where

52%	MATCHING BLOCK 40/64	W

X ij = the number of units shipped from origin i to destination j C ij = cost of shipping a unit from origin i to destination j

S i = supply available at i th origin D j = quantity demanded at j th destination And,

X ij ? 0, for

all i and j Following is the procedure used for solving a transportation problem: 1. Define the objective function that is to be minimized. 2. Develop a

63%	MATCHING BLOCK 41/64	W

transportation table with rows representing the origins and column representing the destinations. 3. Determine the initial feasible solution to the problem. 4. Examine whether the initial solution is feasible or not.

A solution is feasible if the number of occupied cells in the solution is (

90%	MATCHING BLOCK 42/64	W

m + n – 1) where 'm' is the number of origins and 'n' is the number of

90%	MATCHING BLOCK 43/64	W

m + n – 1) where 'm' is the number of origins and 'n' is the number of

90%	MATCHING BLOCK 44/64	W

m + n – 1) where 'm' is the number of origins and 'n' is the number of

destinations. 5.

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Test the solution obtained for optimality by computing the opportunity costs associated with the unoccupied cells. 6. If the solution is not optimum, modify the allocation such that the transportation cost can be reduced further.
Unit 14: Index Numbers 63 15.9.1 Developing an

63%	MATCHING BLOCK 45/64	W

Initial Feasible Solution Following are the methods used for developing an initial feasible solution: North-West Corner Method

In this method, the allocation of products starts at the north-west corner (or the top left corner) of the transportation table. The procedure is given below: 1. Assign the maximum possible quantity of products to the top left corner cell of the transportation problem. 2. After the allocation,

61%	MATCHING BLOCK 46/64	W

adjust the supply and demand numbers. 3. If the supply in the first row is exhausted, move down

69%	MATCHING BLOCK 48/64	W

adjust the supply and demand numbers. 3. If the supply in the first row is exhausted, move down to the corresponding cell in the second row and

assign the possible quantity of products to that cell.

80%	MATCHING BLOCK 49/64	W

If the demand in the column is first satisfied, move horizontally to the next cell in the second column and

assign the quantity of products. 4. Continue the same procedure till the entire requirements are met. 5. Check for feasibility of the solution. Example: Given below is a table showing the distances between a factory and its warehouses and the demand at each warehouse. Find a solution for transporting the goods at the minimum cost for the given transportation problems using the North-West Corner method. Factory/ Warehouse W 1 W 2 W 3 W 4 W 5 Supply F 1 17 7 8 14 11 150 F 2 9 11 12 7 9 250 F 3 13 6 15 10 10 300 Demand 100 120 140 160 180 Solution: Following are the steps involved in solving the given problem using the North-West Corner method: a) Assign the maximum number of goods that can be transported from 'F 1 ' to 'W 1 ', in the cell (F 1 , W 1); i.e. 100. b) Move to the cell (F 1 , W 2) and assign the remaining goods being supplied by F 1 to W 2 ; i.e. 50. c) Move to the cell (F 2 , W 2) and assign the possible number of goods; i.e. 70. d) Move to the cell (F 2 , W 3) and assign the possible number of goods; i.e. 140. e) Move to the cell (F 2 , W 4) and assign the remaining goods being supplied by F 2 to W 4 ; i.e. 40. f) Move to the cell (F 3 , W 4) and assign the possible number of goods; i.e. 120. g) The remaining goods are assigned to the cell (F 3 , W 5); i.e. 180. Factory/ Warehouse W 1 W 2 W 3 W 4 W 5 F 1 100 (17) 50 (7) (8) (14) (11) F 2 (9) 70 (11) 140 (12) 40 (7) (9) F 3 (13) (6) (15) 120 (10) 180 (10)
Block V: Advanced Statistics 64 The solution obtained is feasible as the number of occupied cells is 7, which is equal to the value of (m + n – 1). Transportation cost = = (17×100) + (7 × 50) + (11 × 70) + (12 × 140) + (7 × 40) + (10 × 120) + (10 × 180) = Rs. 7780. Least Cost Method In this method, allocations are made on the basis of unit transportation costs. The following is the procedure: 1. Select

63%	MATCHING BLOCK 50/64	W

the cell with the least unit transportation cost and allocate as many units as possible to that cell. 2. If

the minimum cost exists in several cells, select a cell arbitrarily and assign the possible number of goods. Then consider the remaining cells of the same unit transportation cost. 3. Select a cell with the next higher unit transportation cost and continue the process till all requirements are met. Example: Given below is a table showing the distances between a factory and its warehouses and demand at each warehouse. Find a solution for transporting the goods at the minimum cost for the given transportation problems using the least cost method. Factory/Warehouse W 1 W 2 W 3 W 4 Supply F 1 2 3 11 7 6 F 2 1 0 6 1 1 F 3 5 8 15 9 10 Demand 7 5 3 2 Solution: Following are the steps involved in solving the given problem using the least cost method: a) Consider the cell which has the least unit cost of transportation; i.e. the cell (F 2 , W 2) with a cost of Rs. 0. b) The possible number of goods that can be assigned to the cell (F 2 , W 2) is 1. c) Move to that cell where the next higher unit cost of transportation exists and assign the possible number of goods. d) Continue the process until all the goods have been assigned. Factory/Warehouse W 1 W 2 W 3 W 4 F 1 6 (2) (3) (11) (7) F 2 (1) 1 (0) (6) (1) F 3 1 (5) 4 (8) 3 (15) 2 (9) The solution obtained is feasible as the number of occupied cells is 6, which is equal to the value of (m + n – 1). Transportation cost = (2 × 6) + (5 × 1) + (0 × 1) + (8 × 4) + (15 × 3) + (9 × 2) = Rs. 112.
Unit 14: Index Numbers 65
Vogel's Approximation Method Vogel's Approximation Method is the most preferred method of the three methods as it results in an optimal or a near optimal solution. The following is the procedure: 1. Calculate

79%	MATCHING BLOCK 51/64	W

a penalty for each row and column of the transportation table. The penalty for a row/column is the difference between the least cost and the next least cost

of

87%	MATCHING BLOCK 53/64	W

that row/column. 2. Identify the row or column with the largest penalty

value; and assign the possible quantity of products to the cell with the least unit cost in that row or column. In case of a tie, select the row or column that has minimum cost. 3. Adjust the supply and requirement values after the allocation has been made. 4. Delete that row or column where the supply or requirement is zero. 5. Calculate the values of penalty to all the rows and column for the reduced transportation problem and repeat the procedure till the entire requirement has been met. Example: Given below is a table showing the distances between a factory and its warehouses and the demand at each warehouse. Find the solution for transporting the goods at the minimum cost for the given transportation problems using the Vogel's approximation method. Factory/Warehouse W 1 W 2 W 3 W 4 W 5 Supply F 1 20 28 32 55 70 50 F 2 48 36 40 44 25 100 F 3 35 55 22 45 48 150 Demand 100 70 50 40 40 300 Solution: Following are the steps involved in solving the given problem using the least cost method: a) Compute the

55%	MATCHING BLOCK 54/64	W

penalty for each row and column of the transportation problems. The penalty for the first row is (28 – 20) = 8. Similarly, the

values of penalty for the second and the third row are 11 and 13 respectively. Similarly, the values of penalty for the first, second, third, fourth, and fifth columns are 15, 8, 10, 1, and 23 respectively. b) Identify the row or column with the largest penalty value, i.e., the fifth column with a penalty value of 23. c) The cell with the least cost is chosen and the possible number of goods is assigned to that cell. Therefore, assign 40 to the cell (F 2 , W 5). d) If the remaining row supply or column demand is zero, remove that row/column. e) The process is repeated for the reduced transportation problem till the entire supply at the factories is assigned to satisfy the demand at different warehouses.

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Block V: Advanced Statistics 66 Factory/Warehouse W 1 W 2 W 3 W 4 W 5 F 1 50(20) (28) (32) (55) (70) F 2 (48) 60 (36) (40) (44) 40 (25) F 3 50 (35) 10 (55) 50 (22) 40 (45) (48) The solution obtained is feasible as the number of occupied cells is 7, which is equal to the value of (m + n – 1). Transportation cost = (20 × 50) + (36 × 60) + (25 × 40) + (35 × 50) + (55 × 10) + (22 × 50) + (45 × 40) = Rs. 9,360. Stepping Stone Method After computing the initial solution by using any of the three methods explained, the solution needs to be tested to see whether it is optimum or not by using the stepping stone method. In this method, the decision-maker calculates the net cost change obtained by introducing a unit of quantity in any of the unoccupied cells and checks for the possibility of improving the solution. This method describes the unused cells as 'water' and used cells as 'stones,' and the transportation refers to walking on a path of stones half submerged in the water. The following is the procedure: 1. Determine
the initial basic solution
by using any of the three methods: North-West method, Least Cost method or
the Vogel Approximation method. Check the feasibility of the solution. 2. Select an unoccupied cell and trace a closed path starting from that cell

100%	MATCHING BLOCK 55/64	W
using the most direct route through at least three occupied cells		

by making only horizontal or vertical moves. 3. Starting from the selected cell, assign + and – signs alternatively to the corner cells of the closed path. 4. Calculate the 'net cost change' of the selected cell by adding the unit cost values (with the signs assigned) along the closed path. 5. If the 'net cost change' is positive for all the unoccupied cells, we can conclude that the optimum solution has been arrived at. 6. If the 'net cost change' of an unoccupied cell is negative, the quantity of products to be assigned to that cell is equal to

100%	MATCHING BLOCK 56/64	W
the minimum quantity of those cells with the minus sign in the closed path. 7.		

Repeat the procedure till the optimum solution has been reached. Example: The initial feasible solution of a transportation problem is given below. Using the stepping stone method, test whether the solution is optimum. Calculate the optimum solution if the given solution is not the optimum one. Factory/Warehouse W1 W2 W3 W4 F1 (9) (13) 25(1) 25(6) F2 (12) 60(3) (7) 10(9) F3 30(6) (14) (10) 50(17)
Unit 14: Index Numbers 67 Solution: a) For the unoccupied cell (F 1 , W 1); The closed path is (

100%	MATCHING BLOCK 57/64	W
F 1 , W 1) – (F 1 , W 4) – (F 3 , W 4) – (F 3 , W 1). Net cost change = + 9 – 6 + 17 – 6 = 14 (+		

ve). Therefore, nothing can be assigned to this cell. b) For the unoccupied cell (F 1 , W 2); The closed path is, (

100%	MATCHING BLOCK 58/64	W
F 1 , W 2) – (F 1 , W 4) – (F 2 , W 4) – (F 2 , W 2). Net cost change = + 13 – 6 + 9 – 3 = 13 (+		

ve). Therefore, nothing can be assigned to this cell. c) For the unoccupied cell (F 2 , W 1) ; The closed path is, (

100%	MATCHING BLOCK 59/64	W
F 2 , W 1) – (F 2 , W 4) – (F 3 , W 4) – (F 3 , W 1). Net cost change = + 12 – 9 +17 – 6 = 14 (+		

ve). Therefore, nothing can be assigned to this cell. d) For the unoccupied cell (F 2 , W 3); The closed path is, (

100%	MATCHING BLOCK 60/64	W
F 2 , W 3) – (F 2 , W 4) – (F 1 , W 4) – (F 1 , W 3). Net cost change = + 7 – 9 + 6 – 1 = 3 (+		

ve). Therefore, nothing can be assigned to this cell. e) For the unoccupied cell (F 3 , W 2); The closed path is, (

100%	MATCHING BLOCK 61/64	W
F 3 , W 2) – (F 3 , W 4) – (F 2 , W 4) – (F 2 , W 2). Net cost change = + 14 – 17 + 9 – 3 = 3 (+		

ve). Therefore, nothing can be assigned to this cell. f) For the unoccupied cell (F 3 , W 3); The closed path is, (

100%	MATCHING BLOCK 62/64	W
F 3 , W 3) – (F 3 , W 4) – (F 1 , W 4) – (F 1 , W 3). Net cost change = + 10 – 17 +6 – 1 = - 2 (-		

ve). So, some quantity of products should be assigned to this cell. Let us allocate 25 units to this cell taking it from cell (F 1 , W 3). In the same way, reduce 25 units in cell (F 3 , W 4) and add 25 units to cell (F 1 , W 3). So the transportation table is changed to: Factory/Warehouse W1 W2 W3 W4 Supply F1 (9) (13) (1) 50(6) 50 F2 (12) 60(3) (7) 10(9) 70 F3 30(6) (14) 25 (10) 25(17) 80 Demand 30 60 25 85 200
Block V: Advanced Statistics 68 g) For the unoccupied cell (F 1 , W 3); The closed path is, (

100%	MATCHING BLOCK 63/64	W
F 1 , W 3) – (F 1 , W 4) – (F 3 , W 4) – (F 3 , W 3). Net cost change = + 1 – 6 + 17 – 10 = 2 (+		

ve). Therefore, nothing can be assigned to this cell. Therefore, this is the optimum solution for the given transportation problem.
Check Your Progress - 2 6. In simplex algorithm application, as we move from one table to the other, what will we decide first? a. The variable leaving the solution b. Pivot element c. Pivot row d. Variable entering the solution 7. What are the variables added to the greater than inequalities based on need to ensure the initial solution is feasible for solving through simple approach are referred as? a. Slack variables b. Surplus variables c. Artificial variables d. Basic variables 8. For every LP formulation, we can formulate another unique linear programming formulation, with the data from the original LP formulation, and both these formulations having the same objective value, is called the ________ 9. What is a linear programming problem where one or more decision variables are constrained to take integer values, is known as? a. Goal programming b. Integer programming c. Dynamic programming d. Linear programming 10. In the present setting, profit maximization may not be the only objective of a business concern and there may be other objectives or goals. What is the process when the objectives are weighted and a single objective function is constructed , and is then optimized is known as? a. Goal programming b. Integer programming c. Dynamic programming d. Linear programming

Unit 14: Index Numbers 69 15.10 Summary ? Linear programming technique is useful in solving decision making problems which involve maximizing a linear objective function subject to a set of linear constraints. ? Linear programming is helpful in solving a variety of problems in finance, budgeting and investments, like. o Selection of a product mix maximizing profits o Determination of the capital budget which maximizes the net present value o Choice of mixing short-term financing which minimizes the cost subject to certain funding constraints. ? This chapter covered the basis of linear programming, graphical method of solving the linear programming problem, the simplex method and some elements of post-optimal analysis. ? In arriving at solution through simplex it covered Degeneracy, Alternative Optima, Unbounded Solutions, and Infeasible Solutions. ? It also covered post optimal analysis briefly and duality in details ? It gave introduction of integer programming and goal programming ?
The transportation model is a special case of linear programming and is applied to optimize the distribution system. ? In the transportation model, the initial feasible solution can be developed by using any of the three methods of

87%	MATCHING BLOCK 64/64	W
North-West Corner method, Least cost method, and Vogel's Approximation method. ?		

To verify whether the solution obtained by these three methods is optimal or not, the stepping stone method is used. 15.11
Suggested Readings/Reference Material 1.
Gupta, S. P. Statistical Methods. 46th Revised ed. New Delhi: Sultan Chand & Sons. 2021. 2. I. Levin Richard, H. Siddiqui Masood, S. Rubin David, Rastogi Sanjay. Statistics for Management. Pearson Education; Eighth edition, 2017. 3.
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Arora, P. N., and Arora, S. CA Foundation Course Statistics. 6th ed. S Chand Publishing, 2018. 5.
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David R. Anderson, Dennis J. Sweeney, Thomas A. Williams, Jeffrey D. Camm, James J. Cochran. Statistics for Business and Economics. 13th Edition, Cengage Learning India Pvt. Ltd., 2019.
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Hamdy A. Taha. Operations Research: An Introduction. 10th ed., Pearson, 2016. 9.
Malhotra, N. (2012), Marketing Research: An Applied Orientation, 7
th ed., Pearson, 2019. 10. Cooper, D.R. and Schindler, P.S. and J. K. Sharma (2018), Business Research Methods, 12th edition, McGraw-Hill Education. 15.12
Self-Assessment Questions 1. Explain the linear programming model. 2.
Explain the process of formulating a linear programming problem. 3. What are the different methods of solving a linear programming problem? 4.
Explain the transportation problem of linear programming in detail. 5.
Explain the various methods of developing an initial feasible solution in the transportation method of linear programming. 15.13 Answers to Check Your Progress Questions 1.
Constant. The relationship y = f(x) is defined as linear if, for all possible values of x and y, a given change in the value of x brings about a constant change in the value of y. 2. (c) x is the independent variable, y dependent, a is intercept and b is slope 3. (b) linear constraints. (1) is the objective function and (3) is non negativity restriction 4. (b) Multiple optimal solutions. If the objective function is parallel to an edge of the feasible region, then we get a number points giving the same value for objective function. This is a case of multiple optimal solutions. 5. (a) Minimize - (10x 1 + 21 x 2 + 15 x 3). Just multiply entire equation with minus and change maximization to minimization 6. (d) Variable entering the solution. Then Variable leaving the solution, identify pivot element, compute elements in the next table. 7. (c) Artificial variables. To start within the simplex, when there are greater than equal to inequalities and surplus variables are added to make equations, to get the initial feasible solution values all as positive, we add artificial variables to the Left hand side. 8. 'Dual'. For every LP formulation there exists another unique linear programming formulation called the 'Dual' (the original formulation is called the 'Primal'). Same data can be used for both 'Dual' and
Unit 14: Index Numbers 71 '

Primal' formulation. Both can be solved in a similar manner as the Dual is also an LP formulation. 9. (b) Integer programming. An integer programming problem is identical to a linear programming problem except that one or more decision variables are constrained to take integer values 10. (a) Goal programming. When multiple goals are weighted and a single objective function is constructed to optimize the objective function which is then optimized is known as goal programming

Quantitative Methods Course Structure Block Unit Nos. Unit Title I Introduction to Statistics and Probability 1. Arranging Data 2. Central Tendency and Dispersion 3. Probability 4. Probability Distribution and Decision Theory II Statistical Relations and Hypothesis Testing 5. Statistical Inference and Hypothesis Testing 6. Correlation and Linear Regression III Statistical Regression and Quality Control 7. Multiple Regression 8. Time Series Analysis 9. Quality Control IV Statistical Distributions, Variations and IT 10. Chi-Square Test and Analysis of Variance 11. Role of IT in Modern Business Enterprise 12. Statistical Software Tools V Advanced Statistics 13. Index Numbers 14. Simulation 15. Linear Programming VI Business Research 16. Introduction to Business Research Methods 17. Questionnaire Design 18. Report Writing

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	x 5,000) + (14.00 x 6,000) + (28.00 x 3,000) + (14.50 x 3,000) + (4.50 x 500) = Rs.2,65,000 ?P 1 Q 1 = (10.25 x 4,800) + (14.00 x 5,500) + (28.00 x 3,500) + (14.50 x 2,400) + (4.50 x 600) = Rs.2,61,700 ?P 0 Q 0 = (7.50 x 5,000) + (10 x 6,000) + (25 x 3,000) + (1,060 x 3,000) + (2.60 x 500) = Rs.2,05,600 ?P 0 Q 1 = (7.50 x 4,800) + (10 x 5,500) + (25 x 3,500) + (10.60 x 2,400) + (2.60 x 600) =		x1 + x3 + x5) + 1500 (x2 + x4 + x6) = 10000 x1 + 10000 x3 + 10000 x5 + 2000 x1 + 2000 x3 + 2000 x5 + 1500 x2 + 1500 x4 + 1500 x6 Z = (10000 x1 + 2000 x1) +(1500 x2) + (10000 x3 + 2000x3) + (1500

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2/64	SUBMITTED TEXT 138 WORDS	53%	MATCHING TEXT 138 WORDS
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3/64	SUBMITTED TEXT 138 WORDS	53%	MATCHING TEXT 138 WORDS
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4/64	SUBMITTED TEXT 91 WORDS	44%	MATCHING TEXT 91 WORDS
	x 1 , x 2 , x 3 , ...x n , is y = a 0 + a 1 x 1 + a 2 x 2 + a 3 x 3 +.......+ a n x n where a 0 , a 1 , a 2 , a 3 , ... a n		x = c 1 x 1 + c 2 x 2 + ... +c n x n to restrictions a 11 x 1 + a 12 x 2 + ... + a 1n x n ≤ b 1 a 21 x 1 + a 22 x 2 + ... + a 2n

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5/64	SUBMITTED TEXT 88 WORDS	98%	MATCHING TEXT 88 WORDS
	A 11 x 1 +A 12 x 2 + ... +A 1n x n ≤ b 1 A 21 x 2 +A 22 x 2 +...+A 2n x n ≤ b 2 A m1 x 1 +A m2 x 2 +...A mn x n ≤ b		a 11 X 1 +a 12 X 2 +...+a 1n X n (>,<, ≤, =, ≥) b 1 a 21 X 1 +a 22 X 2 +...+a 2n X n (>,<, ≤, =, ≥) b 2 . . . . A m1 X m +a m2 X 2 +...+a mn X n (>,<, ≤, =, ≥) b

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6/64	SUBMITTED TEXT 88 WORDS	98%	MATCHING TEXT 88 WORDS
	A 11 x 1 +A 12 x 2 + ... +A 1n x n ≤ b 1 A 21 x 2 +A 22 x 2 +...+A 2n x n ≤ b 2 A m1 x 1 +A m2 x 2 +...A mn x n ≤ b		a 11 X 1 +a 12 X 2 +...+a 1n X n (>,<, ≤, =, ≥) b 1 a 21 X 1 +a 22 X 2 +...+a 2n X n (>,<, ≤, =, ≥) b 2 . . . . A m1 X m +a m2 X 2 +...+a mn X n (>,<, ≤, =, ≥) b

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7/64	SUBMITTED TEXT 29 WORDS	45%	MATCHING TEXT 29 WORDS
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8/64 **SUBMITTED TEXT** 106 WORDS **94%** **MATCHING TEXT** 106 WORDS

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A 11 x 1 +A 12 x 2 + ... +A 1n x n ≤ b 1 A 21 x 2 +A 22 x 2 +...+A 2n x n ≤ b 2 A m1 x 1 +A m2 x 2 +...A mn x n ≤ b m x 1 , x 2 , ...x n ? 0.	a 11 x 1 + a 12 x 2 + ... + a 1n x n ≤ b 1 a 21 x 1 + a 22 x 2 + ... + a 2n x n ≤ b 2 . . . a m1 x 1 + a m2 x 2 + ... + a mn x n ≤ b and x 1 ≥ 0, x 2 ≥ 0,..., x n ≥ 0

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9/64 **SUBMITTED TEXT** 49 WORDS **100%** **MATCHING TEXT** 49 WORDS

SUBMITTED TEXT	MATCHING TEXT
Maximize Z = 5x 1 + 7x 2 Subject to 2x 1 + 3x 2 ≤ 13 3x 1 + 2x 2 ≤ 12 x 1 , x 2 ≥0.	Maximize z = 8x1 − 4x2 Subject to, 4x1 + 5x2 ≤ 20 − x1 + 3x2 ≥ − 23 x1 ≥ 0 ; x2

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10/64 **SUBMITTED TEXT** 50 WORDS **96%** **MATCHING TEXT** 50 WORDS

SUBMITTED TEXT	MATCHING TEXT
as: Maximize Z = 5x 1 + 7x 2 Subject to 2x 1 + 3x 2 ≤ 13 3x 1 + 2x 2 ≤ 12 x 1 , x 2 ≥0.	as Maximize (Z) = 50 x 1 +120 x 2 Subject to 2x 1 +4x 2 > 80 3x 1 + X 2 > 60 x 1 , x 2 < 0

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11/64 **SUBMITTED TEXT** 97 WORDS **42%** **MATCHING TEXT** 97 WORDS

SUBMITTED TEXT	MATCHING TEXT
x 1 + 21 x 2 + 15 x 3 to a minimization function a. Minimize -(10x 1 + 21 x 2 + 15 x 3) b. Minimize - 10x 1 + 21 x 2 + 15 x 3 c. Minimize 10x 1 - 21 x 2 + 15 x 3 d. Minimize 10x 1 + 21 x 2 -15 x 3 15.6	x 11 + x 12 + x 13 ≤ 800 (Supply at A) x 21 + x 23 ≤ 1000 (Supply at B) x 31 + x 32 + x 33 ≤ 700 (Supply at C) x 11 + x 21 + x 31 = 900 (Demand at X) x 12 + x 22 +

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12/64 **SUBMITTED TEXT** 98 WORDS **37%** **MATCHING TEXT** 98 WORDS

SUBMITTED TEXT	MATCHING TEXT
x 1 + 21 x 2 + 15 x 3 to a minimization function a. Minimize -(10x 1 + 21 x 2 + 15 x 3) b. Minimize - 10x 1 + 21 x 2 + 15 x 3 c. Minimize 10x 1 - 21 x 2 + 15 x 3 d. Minimize 10x 1 + 21 x 2 -15 x 3 15.6 The	x 2+ 3x 3 - x 4 Subject to: x 1 +2x 2+ 3x 3 =15 2 x 1 +x 2+ 5x 3 =20 x 1 +2x 2+ x 3 + x 4 =10 x 1, x 2, x 3 , x 4 ≥0 the

W http://ndl.ethernet.edu.et/bitstream/123456789/90288/6/Operations%20research%20handout.pdf

13/64	SUBMITTED TEXT	98 WORDS	37%	MATCHING TEXT	98 WORDS

x 1 + 21 x 2 + 15 x 3 to a minimization function a. Minimize -(10x 1 + 21 x 2 + 15 x 3) b. Minimize - 10x 1 + 21 x 2 + 15 x 3 c. Minimize 10x 1 - 21 x 2 + 15 x 3 d. Minimize 10x 1 + 21 x 2 -15 x 3 15.6 The

x 2+ 3x 3 - x 4 Subject to: x 1 +2x 2+ 3x 3 =15 2 x 1 +x 2+ 5x 3 =20 x 1 +2x 2+ x 3 + x 4 =10 x 1, x 2, x 3 , x 4 ≥0 the

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14/64	SUBMITTED TEXT	109 WORDS	98%	MATCHING TEXT	109 WORDS

to the constraints A 11 x 1 +A 12 x 2 + ... +A 1n x n ≤ b 1 A 21 x 2 +A 22 x 2 +...+A 2n x n ≤ b 2 A m1 x 1 +A m2 x 2 +...A mn x n ≤ b m x 1 , x 2 , ...x n ? 0.

SA Optimization Techniques.pdf (D164398531)

15/64	SUBMITTED TEXT	116 WORDS	68%	MATCHING TEXT	116 WORDS

x(2 = 20 0 0.8 0)x(2 1.0 ? = 1.0 0.8 0)x(2 1.0 ? = 1.0 0.8 1.0) x(2 0 ? = −2.5 0.8 0.0) x(2 0 ? = 0 ? (2x0) 0 0.8 = 0 70 q 1 50 0.8 40 ? 1 0.8 0.8 ? ? 0 0 0.8 ? 0 0 0.8 ? 1 1.25 0.8 ? 0 0 0.8 ? 0 0 0.8 0 S L 200 − 0.8 40)x (3.0 = 50 0 2.0 − 0.8 0)x(3 = 2.0 0 − 0.8 0)x(3 = 0 0 − 0.8 1)x(3 = −3.75 1.0 − 0.8 0)x(3 = 1.0 0 − 0.8 0)x(3 = 0 0 S C 360 − 0.8 40) x(4 = 160 0 3.0 − 0.8 0)x(4 = 3.0 0 − 0.8 0)x(4 = 0 0 − 0.8 1)x(4 = −5 0 − 0.8 0)x(4 = 0 1 − 0.8 0)x(4 = 1.0 Z

x1 + 10 x3 + 10 x5) + 2000 (x1 + x3 + x5) + 1500 (x2 + x4 + x6) = 10000 x1 + 10000 x3 + 10000 x5 + 2000 x1 + 2000 x3 + 2000 x2 + 1500 x4 + 1500 Z = (10000

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16/64	SUBMITTED TEXT	116 WORDS	66%	MATCHING TEXT	116 WORDS

x(2 = 20 0 0.8 0)x(2 1.0 ? = 1.0 0.8 0)x(2 1.0 ? = 1.0 0.8 1.0) x(2 0 ? = −2.5 0.8 0.0) x(2 0 ? = 0 ? (2x0) 0 0.8 = 0 70 q 1 50 0.8 40 ? 1 0.8 0.8 ? ? 0 0 0.8 ? 0 0 0.8 ? 1 1.25 0.8 ? 0 0 0.8 ? 0 0 0.8 0 S L 200 − 0.8 40)x (3.0 = 50 0 2.0 − 0.8 0)x(3 = 2.0 0 − 0.8 0)x(3 = 0 0 − 0.8 1)x(3 = −3.75 1.0 − 0.8 0)x(3 = 1.0 0 − 0.8 0)x(3 = 0 0 S C 360 − 0.8 40) x(4 = 160 0 3.0 − 0.8 0)x(4 = 3.0 0 − 0.8 0)x(4 = 0 0 − 0.8 1)x(4 = −5 0 − 0.8 0)x(4 = 0 1 − 0.8 0)x(4 = 1.0 Z

x4 =1 x1 + x2 -x3 +x4 =3 x1, x2 ,x3 ,x4 < 0 -2x1 + x2 +4x3>4 x1 -5 x2 +3x3>1 -3x1 -2x2 +7x3 0 3. Z=7

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17/64	SUBMITTED TEXT 116 WORDS	66% MATCHING TEXT 116 WORDS
	x(2 = 20 0 0.8 0)x(2 1.0 ? = 1.0 0.8 0)x(2 1.0 ? = 1.0 0.8 1.0) x(2 0 ? = −2.5 0.8 0.0) x(2 0 ? = 0 ? (2x0) 0 0.8 = 0 70 q 1 50 0.8 40 ? 1 0.8 0.8 ? ? 0 0 0.8 ? 0 0 0.8 ? 1 1.25 0.8 ? 0 0 0.8 ? 0 0 0.8 0 S L 200 − 0.8 40)x (3.0 = 50 0 2.0 − 0.8 0)x(3 = 2.0 0 − 0.8 0)x(3 = 0 0 − 0.8 1)x(3 = −3.75 1.0 − 0.8 0)x(3 = 1.0 0 − 0.8 0)x(3 = 0 0 S C 360 − 0.8 40) x(4 = 160 0 3.0 − 0.8 0)x(4 = 3.0 0 − 0.8 0)x(4 = 0 0 − 0.8 1)x(4 = −5 0 − 0.8 0)x(4 = 0 1 − 0.8 0)x(4 = 1.0 Z	X2+X3>2 2X1+2X2-X3 <2 3X1 +4X2-X3<8 3X2-4X3<3 X1 , X2, X3< 0 X1+3X3>5 X1 , X2 ,X3< 0 Z=

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18/64	SUBMITTED TEXT 131 WORDS	61% MATCHING TEXT 131 WORDS
	S 1 120 − 0.8 40) x(2 = 20 0 0.8 0)x(2 1.0 ? = 1.0 0.8 0)x(2 1.0 ? = 1.0 0.8 1.0) x(2 0 ? = −2.5 0.8 0.0) x(2 0 ? = 0 ? (2x0) 0 0.8 = 0 70 q 1 50 0.8 40 ? 1 0.8 0.8 ? ? 0 0 0.8 ? 0 0 0.8 ? 1 1.25 0.8 ? 0 0 0.8 ? 0 0 0.8 0 S L 200 − 0.8 40)x (3.0 = 50 0 2.0 − 0.8 0)x(3 = 2.0 0 − 0.8 0)x(3 = 0 0 − 0.8 1)x(3 = −3.75 1.0 − 0.8 0)x(3 = 1.0 0 − 0.8 0)x(3 = 0 0 S C 360 − 0.8 40) x(4 = 160 0 3.0 − 0.8 0)x(4 = 3.0 0 − 0.8 0)x(4 = 0 0 − 0.8 1)x(4 = −5 0 − 0.8 0)x(4 = 0 1 − 0.8 0)x(4 = 1.0 Z j 70 0 0 87.5 0 0	s x x x s x x x s x x x ? ? ? ? ? ? ? ? ? Demand constraints 3 33 23 13 2 32 22 12 1 31 21 11 d x x x x x x d x x ? ? ? ? ? ? ? ? ? j

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19/64	SUBMITTED TEXT 114 WORDS	73% MATCHING TEXT 114 WORDS
	x(2 = 20 0 0.8 0)x(2 1.0 ? = 1.0 0.8 0)x(2 1.0 ? = 1.0 0.8 1.0) x(2 0 ? = −2.5 0.8 0.0) x(2 0 ? = 0 ? (2x0) 0 0.8 = 0 70 q 1 50 0.8 40 ? 1 0.8 0.8 ? ? 0 0 0.8 ? 0 0 0.8 ? 1 1.25 0.8 ? 0 0 0.8 ? 0 0 0.8 0 S L 200 − 0.8 40)x (3.0 = 50 0 2.0 − 0.8 0)x(3 = 2.0 0 − 0.8 0)x(3 = 0 0 − 0.8 1)x(3 = −3.75 1.0 − 0.8 0)x(3 = 1.0 0 − 0.8 0)x(3 = 0 0 S C 360 − 0.8 40) x(4 = 160 0 3.0 − 0.8 0)x(4 = 3.0 0 − 0.8 0)x(4 = 0 0 − 0.8 1)x(4 = −5 0 − 0.8 0)x(4 = 0 1 − 0.8 0)x(4 = 1.0	

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20/64	SUBMITTED TEXT 92 WORDS	97% MATCHING TEXT 92 WORDS
	Maximize 100x + 200y + 0S 1 + 0S 2 + 0S 3 Subject to: 10x + 30y + S 1 = 2,100 10x + 10y + S 2 = 1,000 600x + 800y + S 3 = 72,000 x ? 0, y ? 0, S 1 ? 0, S 2 ? 0, S 3 < 0	Maximize (x + 2y + 0S 1 + 0S 2 + 0S 3 Subject to, 4x + y + S 1 =200 x + y + S 2 = 80 x + 3y + S 3 = 180 x, y, S 1 ,S 2 , S 3 ≥ 0 2.

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21/64 SUBMITTED TEXT 101 WORDS	38% MATCHING TEXT 101 WORDS
Minimize Z = 60q 1 + 80q 2 + 0S 1 + 0S 2 + MA 1 + MA 2 Subject to constraints 20q 1 + 30q 2 − 1S 1 + 0S 2 + 1A 1 + 0A 2 = 900 40q 1 + 30q 2 + 0S 1 − 1S 2 + 0A 1 + 1A 2 = 1200 The initial simplex	Minimize Z=25 x1 +30s1+0s2 +MA1+MA2 Subject to: 20x1+15x2- s1+A1 = 100 2x1+ 3x2 −s2+A2 = 15 x1, x2 , s1, s2 ,A1 ,A2 < 0 37 Step 2 Initial simplex

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22/64 SUBMITTED TEXT 101 WORDS	38% MATCHING TEXT 101 WORDS
Minimize Z = 60q 1 + 80q 2 + 0S 1 + 0S 2 + MA 1 + MA 2 Subject to constraints 20q 1 + 30q 2 − 1S 1 + 0S 2 + 1A 1 + 0A 2 = 900 40q 1 + 30q 2 + 0S 1 − 1S 2 + 0A 1 + 1A 2 = 1200 The initial simplex	Minimize Z=25 x1 +30s1+0s2 +MA1+MA2 Subject to: 20x1+15x2- s1+A1 = 100 2x1+ 3x2 −s2+A2 = 15 40 x1, x2 , s1, s2 ,A1 ,A2 < 0 Step 2 Initial simplex

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23/64 SUBMITTED TEXT 101 WORDS	38% MATCHING TEXT 101 WORDS
Minimize Z = 60q 1 + 80q 2 + 0S 1 + 0S 2 + MA 1 + MA 2 Subject to constraints 20q 1 + 30q 2 − 1S 1 + 0S 2 + 1A 1 + 0A 2 = 900 40q 1 + 30q 2 + 0S 1 − 1S 2 + 0A 1 + 1A 2 = 1200 The initial simplex	Minimize Z=25 x1 +30s1+0s2 +MA1+MA2 Subject to: 20x1+15x2- s1+A1 = 100 2x1+ 3x2 −s2+A2 = 15 x1, x2 , s1, s2 ,A1 ,A2 < 0 37 Step 2 Initial simplex

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24/64 SUBMITTED TEXT 52 WORDS	73% MATCHING TEXT 52 WORDS
M 900 20 30 −1 0 1 0 900/20 A 2 M 1200 40 30 0 −1 0 1 1200/40 Z j 60M 60M −M −M M M C j − Z j 60− 60M 80− 60M M M 0 0	M A 1 6 8 -1 0 1 0 100 25/2 M A 2 7 (12) 0 -1 0 1 120 10 Z j =∑ C B a ij 13M 20M -M -M M M 220M ↑I.S Δj = Z j - Cj 13M-12 20M-20 -M -M 0 0 ↑

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25/64 SUBMITTED TEXT 52 WORDS	73% MATCHING TEXT 52 WORDS
M 900 20 30 −1 0 1 0 900/20 A 2 M 1200 40 30 0 −1 0 1 1200/40 Z j 60M 60M −M −M M M C j − Z j 60− 60M 80− 60M M M 0 0	M A 1 6 8 -1 0 1 0 100 25/2 M A 2 7 (12) 0 -1 0 1 120 10 Z j =∑ C B a ij 13M 20M -M -M M M 220M ↑I.S Δj = Z j - Cj 13M-12 20M-20 -M -M 0 0 ↑

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26/64	SUBMITTED TEXT	57 WORDS	47%	MATCHING TEXT	57 WORDS

A 1 M 900 20 30 −1 0 1 0 900/20 A 2 M 1200 40 30 0 −1 0 1 1200/40 Z j 60M 60M −M −M M M C j − Z j 60− 60M 80− 60M M M 0 0 Key Column

A1 2 1 1 1 1 −1 0 1 0 M A2 1 2 1 −1 −2 0 −1 0 1 3M 2 M 0 −M −M −M M M 2 M Zj Cj − Zj 10 − 3M 6 − 2M M M 0 Minimum ratio = key column 2/1 = 2 1/2 = 0.5 → 0 ↑

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27/64	SUBMITTED TEXT	65 WORDS	63%	MATCHING TEXT	65 WORDS

A 1 M 300 0 15 −1 1/2 1 −1/2 20 q 1 60 30 1 3/4 0 −1/40 0 1/40 40 Z j 60 15M+ 45 −M M/2 − 3/2 M −M/2 + 3/2 C j − Z j 0 35− 15M M 3/2− M/2 0 − 3 2 + 3M 2

A 1 6 8 -1 0 1 0 100 25/2 M A 2 7 (12) 0 -1 0 1 120 10 Z j =∑ C B a ij 13M 20M -M -M M M 220M ↑I.S ∆j = Z j - Cj 13M-12 20M-20 -M -M 0 0 ↑

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28/64	SUBMITTED TEXT	65 WORDS	63%	MATCHING TEXT	65 WORDS

A 1 M 300 0 15 −1 1/2 1 −1/2 20 q 1 60 30 1 3/4 0 −1/40 0 1/40 40 Z j 60 15M+ 45 −M M/2 − 3/2 M −M/2 + 3/2 C j − Z j 0 35− 15M M 3/2− M/2 0 − 3 2 + 3M 2

A 1 6 8 -1 0 1 0 100 25/2 M A 2 7 (12) 0 -1 0 1 120 10 Z j =∑ C B a ij 13M 20M -M -M M M 220M ↑I.S ∆j = Z j - Cj 13M-12 20M-20 -M -M 0 0 ↑

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29/64	SUBMITTED TEXT	21 WORDS	68%	MATCHING TEXT	21 WORDS

a company produces two products (A and B) on two different machines. One unit of product A takes 3 hours

A company produces two products A and B on two machines. A unit of product A requires 2 hours

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30/64	SUBMITTED TEXT	20 WORDS	50%	MATCHING TEXT	20 WORDS

constraints. The coefficients in the Primal objective function become the right-hand-side constraints in the Dual constraints. The column of

constraints become the objective function coefficients in the dual with new decision variables. 5. The objective function coefficients become the right hand side values of constraints in the dual. Activity 2.6: State the dual of

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31/64	SUBMITTED TEXT 56 WORDS	100% MATCHING TEXT 56 WORDS
	y 1 + 14y 2 subject to 2y 1 + 2y 2 ? 12 3y 1 + y 2 ??10 y 1 ?? 0, y 2 <??0	Y2 + 25Y3, subject to: 4Y1 + 5Y2 + 6Y3 ≥ 5, 7Y1 + 2Y2 + 8Y3 ≥ 6,

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32/64	SUBMITTED TEXT 152 WORDS	56% MATCHING TEXT 152 WORDS
	x 1 + 10x 2 subject to 2x 1 + 3x 2 >?18 2x 1 + x 2 ??14 x 1 , x 2 < 0 The 'Dual' formulation for this problem would be Minimize 18y 1 + 14y 2 subject to 2y 1 + 2y 2 ? 12 3y 1 + y 2 ??10 y 1 ?? 0, y 2 <??0	x 1 + 30 x 2 Subject to x 1 + 3x 2 > 46 2x 1 - 5x 2 = 60 3x 2 < 30 x 1, x 1 < 0 Answer of Minimize = 120 y 2 + 360 y 3 Subject to y 1 + 2y 2 + y 3 ≥ 30 y 1 + 3y 2 + 4y 3 ≥ 40

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33/64	SUBMITTED TEXT 58 WORDS	95% MATCHING TEXT 58 WORDS
	Minimize 18y 1 + 14y 2 subject to 2y 1 + 2y 2 ? 12 3y 1 + y 2 ??10 y 1 ?? 0, y 2 <??0	Minimize 2 1 2 3 y W ? ? Subject to 0 ,0 ,3 2 ,6 3 2 1 2 1 2 1 ? ? ? ? ? ? y y y y y y

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34/64	SUBMITTED TEXT 114 WORDS	38% MATCHING TEXT 114 WORDS
	Primal Dual Minimize Z Maximize Z = 60q 1 + 80q 2 = 900y 1 + 1200y 2 Subject to Subject to 20q 1 + 30q 2 < 900 20y 1 + 40y 2 > 60 40q 1 + 30q 2 < 1200 30y 1 + 30y 2 > 80 q 1 , q 2 < 0 y 1 , y 2 < 0	Primal Dual Max Z = 2x 1 +3x 2 Min Z' = 18y 1 +19y 2 Subject to: Subject to: 4x 1 + 3x 2 ≤ 18, 4y 1 + 5y 2 ≥ 2, 5x 1 +2x 2 ≤ 19, 3y 1 + 2y 2 ≥ 3, x 1 , x 2 ≥ 0. y 1 , y 2 ≥ 0.

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35/64	SUBMITTED TEXT 114 WORDS	38% MATCHING TEXT 114 WORDS
	Primal Dual Minimize Z Maximize Z = 60q 1 + 80q 2 = 900y 1 + 1200y 2 Subject to Subject to 20q 1 + 30q 2 < 900 20y 1 + 40y 2 > 60 40q 1 + 30q 2 < 1200 30y 1 + 30y 2 > 80 q 1 , q 2 < 0 y 1 , y 2 < 0	Primal Dual Max Z = 2x 1 +3x 2 Min Z' = 18y 1 +19y 2 Subject to: Subject to: 4x 1 + 3x 2 ≤ 18, 4y 1 + 5y 2 ≥ 2, 5x 1 +2x 2 ≤ 19, 3y 1 + 2y 2 ≥ 3, x 1 , x 2 ≥ 0. y 1 , y 2 ≥ 0.

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36/64	SUBMITTED TEXT	73 WORDS	55%	MATCHING TEXT	73 WORDS

Maximize Z = 900y 1 + 1200y 2 + 0S 1 + 0S 2 Subject to 20y 1 + 40y 2 + 1S 1 + 0S 2 = 60 30y 1 + 30y 2 + 0S 1 + 1S 2 = 80 In the initial solution

Maximize (z) = 50 Y + 0 S 1 + 0S 2 Subject to, 2 X + 3Y + S 1 =1500 3 X + 2 Y + S 2 = 1500 X, S 1 ,Find the optimal solution

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37/64	SUBMITTED TEXT	12 WORDS	100%	MATCHING TEXT	12 WORDS

from plants to warehouses, warehouses to wholesalers, wholesalers to retailers, and

from plants to warehouses, warehouses to wholesalers, wholesalers to retailers and

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38/64	SUBMITTED TEXT	20 WORDS	91%	MATCHING TEXT	20 WORDS

physical movement of goods from plants to warehouses, warehouses to wholesalers, wholesalers to retailers, and from retailers to

physical movement of goods from plants to warehouses, warehouses to wholesalers, wholesalers to retailers, and retailers to

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39/64	SUBMITTED TEXT	29 WORDS	38%	MATCHING TEXT	29 WORDS

be given as Minimize ? ? ij X ij C Z Subject to the supply constraints, 2...m 1, i and S X i n 1j ij ? ? ? ? Demand constraints, 2...n 1, j and D X j m 1i ij ? ? ? ?

be given as follows: ?? ? ? ? m n j ij ij xc Z Minimize 1 1 Subject to the constraints int) (,...,2,1 ; 1 constra capacity m is x i n j ij ? ? ? ? . int) (,...,2,1 ; 1 constra t requiremen n j d x j m i ij ? ? ? ?

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40/64	SUBMITTED TEXT	33 WORDS	52%	MATCHING TEXT	33 WORDS

X ij = the number of units shipped from origin i to destination j C ij = cost of shipping a unit from origin i to destination j

X ij = the quantity to be transported from origin I to destination j C ij = cost of transporting one unit of product from origin I to destination j

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41/64	SUBMITTED TEXT	33 WORDS	63% MATCHING TEXT	33 WORDS
transportation table with rows representing the origins and column representing the destinations. 3. Determine the initial feasible solution to the problem. 4. Examine whether the initial solution is feasible or not.			transportation table with m rows representing the sources (plants, factories etc.) and n columns representing the destinations (warehouses, stores, markets etc). 3. Develop an initial feasible solution to the problem 4. Examine whether the initial solution is feasible or not.	

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42/64	SUBMITTED TEXT	19 WORDS	90% MATCHING TEXT	19 WORDS
m + n – 1) where 'm' is the number of origins and 'n' is the number of			m + n – 1) where, 'm' is the number of rows and 'n' is the number of	

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43/64	SUBMITTED TEXT	19 WORDS	90% MATCHING TEXT	19 WORDS
m + n – 1) where 'm' is the number of origins and 'n' is the number of			m+n-1 where m is the number of rows and n is the number of	

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44/64	SUBMITTED TEXT	19 WORDS	90% MATCHING TEXT	19 WORDS
m + n – 1) where 'm' is the number of origins and 'n' is the number of			m+n-1 where m is the number of rows and n is the number of	

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45/64	SUBMITTED TEXT	21 WORDS	63% MATCHING TEXT	21 WORDS
Initial Feasible Solution Following are the methods used for developing an initial feasible solution: North-West Corner Method			Initial Feasible Solution The following are the methods of finding initial feasible solution, 1. North West Corner Method (	

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46/64	SUBMITTED TEXT	20 WORDS	61%	MATCHING TEXT	20 WORDS

adjust the supply and demand numbers. 3. If the supply in the first row is exhausted, move down

Adjust the supply and demand as per the allocations made. 3. (i) If the supply for the source (row) is exhausted, then move vertically down

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47/64	SUBMITTED TEXT	23 WORDS	64%	MATCHING TEXT	23 WORDS

of the dual variables (7/3, 1/3) are the coefficients of the slack variables in the ? j row of the

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48/64	SUBMITTED TEXT	29 WORDS	69%	MATCHING TEXT	29 WORDS

adjust the supply and demand numbers. 3. If the supply in the first row is exhausted, move down to the corresponding cell in the second row and

Adjust the supply and demand numbers in the respective rows and columns allocations. iii) If: a) The supply for first row is exhausted, then move down to the first cell in the second row and

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49/64	SUBMITTED TEXT	21 WORDS	80%	MATCHING TEXT	21 WORDS

If the demand in the column is first satisfied, move horizontally to the next cell in the second column and

If the demand for the first column is satisfied then move horizontally to the next cell in the second column and

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50/64	SUBMITTED TEXT	21 WORDS	63%	MATCHING TEXT	21 WORDS

the cell with the least unit transportation cost and allocate as many units as possible to that cell. 2. If

The cell with the smallest unit cost of transportation is choosen and as many units as possible are allocated to that cell. If

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51/64	SUBMITTED TEXT	29 WORDS	79%	MATCHING TEXT	29 WORDS

a penalty for each row and column of the transportation table. The penalty for a row/column is the difference between the least cost and the next least cost

a penalty for each row and column in the transportation table. The penalty for a given row and column is merely the difference between the smallest cost and the next smallest cost

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52/64	SUBMITTED TEXT	60 WORDS	84%	MATCHING TEXT	60 WORDS

y 1 + 1200y 2 + 0S 1 + 0S 2 Subject to 20y 1 + 40y 2 + 1S 1 + 0S 2 = 60 30y 1 + 30y 2 + 0S 1 + 1

SA operations research finnal.docx (D126058820)

53/64	SUBMITTED TEXT	14 WORDS	87%	MATCHING TEXT	14 WORDS

that row/column. 2. Identify the row or column with the largest penalty

that particular row or column. ii) Identify the row or column with the largest penalty.

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54/64	SUBMITTED TEXT	26 WORDS	55%	MATCHING TEXT	26 WORDS

penalty for each row and column of the transportation problems. The penalty for the first row is (28 – 20) = 8. Similarly, the

penalty for each row and column in the transportation table. The penalty for a given row and column is merely the

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55/64	SUBMITTED TEXT	12 WORDS	100%	MATCHING TEXT	12 WORDS

using the most direct route through at least three occupied cells

using the most direct route through at least three occupied cells

W https://elearning.daystar.ac.ke/pluginfile.php?forcedownload=1&file=%2F39647%2Fcourse%2Foverviewf ...

56/64	SUBMITTED TEXT	17 WORDS	100%	MATCHING TEXT	17 WORDS

the minimum quantity of those cells with the minus sign in the closed path. 7.

the minimum quantity of those cells with the minus sign in the closed path

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57/64	SUBMITTED TEXT 59 WORDS	100% MATCHING TEXT 59 WORDS
	F 1 , W 1) – (F 1 , W 4) – (F 3 , W 4) – (F 3 , W 1). Net cost change = + 9 – 6 + 17 – 6 = 14 (+	F 1 W 1) +1 +21 (F 1 W 4) -1 -13 (F 3 W 4) +1 +41 (F 3 W 1) -1 -32 Net Cost Change +17

W https://elearning.daystar.ac.ke/pluginfile.php?forcedownload=1&file=%2F39647%2Fcourse%2Foverviewf ...

58/64	SUBMITTED TEXT 59 WORDS	100% MATCHING TEXT 59 WORDS
	F 1 , W 2) – (F 1 , W 4) – (F 2 , W 4) – (F 2 , W 2). Net cost change = + 13 – 6 + 9 – 3 = 13 (+	F 1 W 1) +1 +21 (F 1 W 4) -1 -13 (F 3 W 4) +1 +41 (F 3 W 1) -1 -32 Net Cost Change +17

W https://elearning.daystar.ac.ke/pluginfile.php?forcedownload=1&file=%2F39647%2Fcourse%2Foverviewf ...

59/64	SUBMITTED TEXT 58 WORDS	100% MATCHING TEXT 58 WORDS
	F 2 , W 1) – (F 2 , W 4) – (F 3 , W 4) – (F 3 , W 1). Net cost change = + 12 – 9 +17 – 6 = 14 (+	F 1 W 1) +1 +21 (F 1 W 4) -1 -13 (F 3 W 4) +1 +41 (F 3 W 1) -1 -32 Net Cost Change +17

W https://elearning.daystar.ac.ke/pluginfile.php?forcedownload=1&file=%2F39647%2Fcourse%2Foverviewf ...

60/64	SUBMITTED TEXT 59 WORDS	100% MATCHING TEXT 59 WORDS
	F 2 , W 3) – (F 2 , W 4) – (F 1 , W 4) – (F 1 , W 3). Net cost change = + 7 – 9 + 6 – 1 = 3 (+	F 1 W 1) +1 +21 (F 1 W 4) -1 -13 (F 3 W 4) +1 +41 (F 3 W 1) -1 -32 Net Cost Change +17

W https://elearning.daystar.ac.ke/pluginfile.php?forcedownload=1&file=%2F39647%2Fcourse%2Foverviewf ...

61/64	SUBMITTED TEXT 59 WORDS	100% MATCHING TEXT 59 WORDS
	F 3 , W 2) – (F 3 , W 4) – (F 2 , W 4) – (F 2 , W 2). Net cost change = + 14 – 17 + 9 – 3 = 3 (+	F 1 W 1) +1 +21 (F 1 W 4) -1 -13 (F 3 W 4) +1 +41 (F 3 W 1) -1 -32 Net Cost Change +17

W https://elearning.daystar.ac.ke/pluginfile.php?forcedownload=1&file=%2F39647%2Fcourse%2Foverviewf ...

62/64	SUBMITTED TEXT 59 WORDS	100% MATCHING TEXT 59 WORDS
	F 3 , W 3) – (F 3 , W 4) – (F 1 , W 4) – (F 1 , W 3). Net cost change = + 10 – 17 +6 – 1 = - 2 (-	F 1 W 1) +1 +21 (F 1 W 4) -1 -13 (F 3 W 4) +1 +41 (F 3 W 1) -1 -32 Net Cost Change +17

W https://elearning.daystar.ac.ke/pluginfile.php?forcedownload=1&file=%2F39647%2Fcourse%2Foverviewf ...

Ouriginal by Turnitin

63/64	SUBMITTED TEXT	59 WORDS	100%	MATCHING TEXT	59 WORDS

F 1 , W 3) – (F 1 , W 4) – (F 3 , W 4) – (F 3 , W 3). Net cost change = + 1 – 6 + 17 – 10 = 2 (+

F 1 W 1) +1 +21 (F 1 W 4) -1 -13 (F 3 W 4) +1 +41 (F 3 W 1) -1 -32 Net Cost Change +17

W https://elearning.daystar.ac.ke/pluginfile.php?forcedownload=1&file=%2F39647%2Fcourse%2Foverviewf ...

64/64	SUBMITTED TEXT	13 WORDS	87%	MATCHING TEXT	13 WORDS

North-West Corner method, Least cost method, and Vogel's Approximation method. ?

North West Corner method, Least Cost Method, and Vagel's Approximation Method

W https://vle.seu.ac.lk/mod/resource/view.php?id=32018